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Models for the Optimization of Air Refueling Missions

THESIS

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THESIS

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Requirements for the Degree of
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Clayton Hugh Pflieger

Table of Contents

List of Figures

List of Tables

Abstract

The purpose of this research was to follow the work done by A. Yamani in the area of minimizing the total cost of an airlift mission that requires in-flight refueling through the selection of the rendezvous point and the initial fuel for each aircraft. During this effort, Yamani's major assumptions were removed and the resulting enhanced formulation was applied the the problem of minimizing the total fuel cost of a mission and to the related problem of maximizing the allowable cabin load of the airlifter.

The formulation considers a single C-141B airlifter that is to be refueled once by a single KC-135E tanker at some point enroute to the airlifter's destination. The decision variables are the latitude and longitude coordinates of the rendezvous point as well as the initial fuel of each aircraft. Aircraft flight is assumed to take place at a constant altitude along the great-circle arc connecting the points in question. The fuel costs of climb and cruise flight are modeled as functions of the aircraft gross weight, while the fuel costs associated with the air refueling maneuver, the descent, landing, and required reserve are modeled as constant numbers. A 200 nautical mile air refueling track is represented in the model along with the effects of a constant wind over the route of flight. The necessity of a refueling alternate for the airlifter is also included.

The method of Sequential Quadratic Programming was used to obtain numerical results for both applications of the model. Comparison of model results with a computer-generated flight plan shows that substantial fuel savings and a large increase of allowable cabin load are possible through the selection of an optimal rendezvous point and initial fuel combination. However, the models must be applied carefully. Minimization of the mission fuel cost is only appropriate in situations where the cargo weight is a fixed quantity less than the maximum capacity of the

aircraft. Otherwise, the cargo load should be maximized as this reduces the total number of missions required to move the cargo and thus provides the greatest savings.

Additional research is recommended for further enhancement of the model and for its application to areas such as tanker basing and deployment.

Models for the Optimization of Air Refueling Missions

I. Introduction

1.1 Overall Problem

Perhaps the greatest challenge currently facing the Air Force is how to preserve a variety of mission capabilities while simultaneously cutting cost. The only way to lessen the sacrifice necessary to meet budget goals is to operate more efficiently. One way this can be done is through the frugal use of resources such as aviation fuel. According to Air Force Regulation 60-16:

> It is Air Force policy to conserve aviation fuel when it does not adversely affect training, flight safety, or operational readiness [11:4].

One type of flying operation that involves large quantities of fuel is Air Refueling (AR). However, air refueling missions are not planned as efficiently as possible. The problem of creating an optimal air refueling flight plan is complicated by the interrelationships that exist between parameters such as aircraft gross weight, wind, rendezvous location, and initial fuel load. Currently, there is no means other than experience and heuristics for optimizing AR flight plans [4]. Because of this, AR missions are planned as feasible, rather than optimal solutions. If the total fuel cost of an AR mission could be minimized by selecting the optimal rendezvous location and initial fuel for each aircraft, then measurable savings would be gained without sacrifice. Such optimization is the general issue behind this research.

1.2 Background

Air refueling is not done for the sake of efficiency. In fact, it is always more costly than landing enroute for fuel [4]. Instead, AR extends the range of an aircraft so that a bomber may strike a more distant target, a fighter may provide greater coverage, or a transport can reach a more distant location than would otherwise be possible. This research is concerned with air refueling in the context of airlift missions. For these missions, AR becomes a critical factor when supplies must be delivered to a remote location and forward bases are not available. Overall, the AR capability is a vital asset to the Air Force, and its fuel intensive nature makes AR a good candidate for cost-saving optimization.

In order to be useful, such optimization must be applied to the process of flight planning AR missions. Mission planners at headquarters are responsible for providing the flight plans for airlift missions including those that require air refueling [5]. These flight plans determine how much cargo can be carried and the mission cost in terms of time and fuel. Even if the mission is not flown according to plan, the aircraft must still depart with the prescribed initial fuel and cargo load. Therefore the mission planning is critical to how effective the mission will be in terms of cargo delivered and efficiency.

The mission planner's problem may be presented in one of two ways [4]:

- An aircraft hauls a given amount of cargo from point A to point B, how can the mission be planned to minimize the fuel cost?
- Cargo must be flown from point A to point B. How can the amount of cargo per sortie be maximized?

When the distance between point A and point B is short enough to allow the flight to be done without air refueling, then the optimization of the flight plan is fairly simple and straightforward. When air refueling is required, the problem of minimizing cost or maximizing cargo weight still exists, except now there is another aircraft to consider. The flight planner must determine the route of flight to include an air refueling point, as well as the initial fuel load of the airlifter. Another headquarters flight planner develops a flight plan for the tanker that supports the airlift mission [4]. This process does not, in general, provide the most efficient missions in terms of total fuel cost to the Air Force. The reason for this is twofold. First, the mission planning is one-sided in favor of the receiver aircraft. Second, flight planners have only heuristic means of selecting the refueling point, cargo load, and initial fuel load. The problem of determining the optimal combination of these quantities is too difficult to be solved manually or by heuristic, therefore it is a good candidate for mathematical programming.

In 1986, Abdulrahman Yamani published his doctoral dissertation, *Analysis of an Air Transportation System.* In his dissertation, Yamani formulated and solved the flight planner's problem as a Non-Linear Program (NLP). Unfortunately, Yamani's formulation lacks operational "flavor". It contains too many simplifying assumptions that must be resolved before a real conclusion about the usefulness of NLP as a realistic solution to the flight planner's problem can be reached.

1.3 Problem Statement and Approach

The purpose of this research is to extend Yamani's formulation by removing the major simplifying assumptions and thereby create an analysis tool that can be used to solve the AR mission planning problem. In order to do this, the bulk of Yamani's assumptions are removed through an incremental process of model enhancement thereby extending Yamani's work by the following steps:

1. Yamani's formulation is solved in order to verify the solution method.
2. Data is taken from the KC-135E and C-141B manuals and applied to a realistic mission profile.

3. Costs of takeoff, climb, descent, landing, reserve fuel, and the necessity of a refueling alternate for the receiver aircraft are included.
4. A nonzero refueling distance and the additional fuel costs of the AR maneuver are accounted for.
5. The effects of wind are included.
6. The last model, which includes all of the enhancements, is modified to determine the maximum cargo weight that may be carried on a particular mission.

1.4 Methodology

The method of solution to this problem is one of formulating and solving NLP models both for Yamani's formulation and the enhancements that follow. However, the reader may question the use of non-linear programming as a solution method. This is a valid concern because non-linear programming is considered to be difficult. However, embedded in the nature of Yamani's formulation is the fact that changes in the decision variables do not produce proportional changes in the objective function and the effect of one variable is dependent on the values of the others. This results from both the dependency of fuel consumption on the amount of fuel present, and the way distances are measured along the surface of a sphere. The reasons for this are apparent when the model in section 3.2, but these characteristics violate the proportionality and additivity assumptions necessary for the problem to be considered a Linear Programming (LP) problem [25:52]. Since the LP assumptions do not hold due to the nature of the functions involved NLP methods are the only means available to solve the problem. As shown in this research, Yamani's formulation, and the models based on it, can be solved by various NLP methods without undue difficulty.

1.5 Assumptions

Because all models must, by definition, be abstractions of reality, they all have associated assumptions. A number of assumptions are applied throughout the modeling process to help with various details. These are explained as they occur, but the significant assumptions that are common to all models in this research are listed here:

- All locations are considered to be points on the surface of a sphere with the same radius as the Earth.
- All distances are measured along the great-circle arc connecting the points in question.
- The aircraft are free to follow great-circle routes and do not have to adhere to a particular route structure.
- Overflight of any location is permitted.
- Cruise flight, and the air refueling itself takes place at an altitude of 31,000 feet.
- No alternate destination is required due to weather for either aircraft.
- Crew duty day restrictions are not considered.

Due to the air route structure, it is not possible to fly along great-circle arcs in general. However, it is possible to approximate them. Long distance flights do not take place at a constant altitude in general; higher altitudes are preferred because aircraft fuel mileage increases with altitude [5]. The assumed altitude of 31,000 feet may be considered a medium altitude, so it is chosen as a compromise over the more difficult alternative of modeling a multi-altitude mission.

1.6 Summary

The need exists for this type of model, and Yamani has laid a tremendous groundwork for it. This research shows that non-linear programs based on Yamani's formulation are viable analysis tools that can solve the AR mission planning problem. The next chapter covers some important background information as well as a brief description of the work done by all known investigators of the AR mission planning problem. Chapter 3 is devoted to the model formulations and Chapter 4 covers the results obtained from them. The fifth and final chapter gives the conclusions of this effort and the recommendations for further research.

II. Literature Review

2.1 Fuel Requirements

Air Force Regulation 60-16, *General Flight Rules*, provides the fundamental rules for the operation of all Air Force aircraft. Although broad in scope, AFR 60-16 prescribes specific rules for fuel requirements. According to the regulation,

> Before takeoff or immediately after in-flight refueling, there must be enough useable fuel aboard the aircraft to complete the flight:
>
> 1. To a final landing, either at the destination airport or alternate airport (if one is required), plus the fuel reserve.
> 2. To or between air refueling control points (ARCP) and then to land at the destination (or a recovery base, if refueling is not successful), plus the fuel reserve [11:6].

The primary reason for this is safety. An aircrew should never be forced into a dangerous situation because the weather was bad at the intended destination or an air refueling was unsuccessful. In order to ensure the AFR 60-16 fuel requirements are met, a flight plan is used for every flight of an Air Force aircraft [11:9].

2.2 Mission Planning

Mission planners at headquarters are responsible for providing the flight plans for airlift missions, including both air refueling and non-AR flights [5]. When the distance involved is short enough that AR is not required, the optimization of the flight plan is fairly simple and straightforward. The route of flight is chosen to be as close as possible to the great-circle arc connecting the origin and the destination [5]. Once the route of flight is determined, the fuel planning is done in reverse. By starting with the aircraft on the ground with the required reserve at the primary or alternate destination and then working backward through the flight, the fuel required at each point, and thus the whole flight, can be minimized [17]. If more fuel than necessary is carried, then about three percent of this excess is used to carry the fuel as "cargo" [5]. Every pound of unnecessarily transported fuel represents a pound of cargo that could have been carried on the mission [4]. Either way, there is an incentive for minimizing the initial fuel down to the limits imposed by safety. Reverse flight planning finds this minimum exactly for the case of one aircraft in unrefueled flight [4].

When AR is required, the flight planner must determine the route of flight to include an air refueling track as well as the initial fuel [17]. The most desired location for an AR track is one that is close to the airlifter's route of flight, close to the tanker base, and offers the most efficient abort to the alternate [17]. Typically, published AR tracks are used because there are many available and coordination with Air Traffic Control (ATC) is much easier than it is when attempting to created one for a specific mission [17].

Once the flight planner has the route of flight and AR track in mind, the initial fuel load for the airlifter is determined. Normally, the initial fuel is set to the minimum fuel necessary to fly from the origin to the refueling point and to the alternate [4]. The tanker is expected to supply the rest of the fuel. Next, the airlift flight planner contacts the person responsible for tanker flight planning to coordinate the mission [4]. The flight planners responsible for tanker flights work at another headquarters organization and they determine if the requested amount of fuel can be delivered where and when the airlifter needs it [4]. If it is possible, then the mission is planned and flown. If the tanker cannot deliver the full amount, then the airlift planner increases the initial fuel load or reduces the cargo weight so that the onload requirement is reduced to what the tanker can supply [17].

When AR mission planning is done by current methods, the location of the AR track is determined by experience, heuristic, and published track availability. Also, the practice of taking off with just enough fuel to reach the refueling track and then the alternate forces the tanker to supply more fuel than it would if the receiver had taken off heavier. This typically increases the total fuel cost [3:57]. Admittedly, the method does not minimize fuel consumption for a given mission [17]. Emphasis is placed on maximizing payload and producing feasible flight plans within the time available [17].

2.3 Investigations of the Mission Planning problem

2.3.1 Bordelon and Marcotte. The general problem of minimizing the total fuel cost in an air refueling mission has been investigated previously. Two of the investigators were Bordelon and Marcotte, who wrote a joint thesis in 1981 titled *Optimization of Strategic Airlift In-Flight Refueling.* Their stated primary objective was the following:

> ...to develop a method which determines the combination of in-flight refueling rendezvous point, takeoff fuel loads, and tanker base which results in the minimum total fuel consumption for an airlifter and tanker aircraft [3:1-2].

In order to meet this objective, they built an analytic model to find the optimal rendezvous location and takeoff fuels, and a stochastic model to test the feasibility of these results. They expected their models to verify the following two hypothesis:

1. The minimum total fuel consumed by the airlift and tanker aircraft for their combined flight will result from a rendezvous point located at the maximum flight range of the airlifter from its destination base. This point is always located on the boundary of the region of feasible rendezvous points closest to the airlifter takeoff base [3:2].
2. Airlifter aircraft departures with the maximum allowable fuel load will always result in the minimum total fuel consumption for both aircraft. This implies that the fuel transferred is the minimum required to complete the flight [3:2].

Their analytic model was called FLTPLN, and it was used to find the optimal rendezvous point and initial fuel for the mission [3:3]. It begins by assigning the takeoff fuel of the receiver aircraft to its maximum value [3:167]. The rendezvous point is set to be 250 nautical miles short of the point along the great-circle arc from the departure base to the destination where unrefueled flight becomes possible

[3:167]. The 250 nm distance comes from the assumption that the refueling track extends for 250 miles along the route of flight for the receiver aircraft [3:167]. Next, a subroutine searches a database to obtain a set of constants that define the sixth-degree polynomial approximations to the fuel functions [3:169]. These fuel functions are used to compute the minimum fuel required to complete the mission given the current rendezvous point and takeoff fuels. Then the rendezvous point is iterated through a rectangle of 20 degrees of latitude in increments of five, and 24 degrees of longitude in increments of two, with the total fuel consumption computed for each intersection [3:171]. At each step, a feasibility check is performed and infeasible points are given a flag value [3:171]. The resulting 5 by 13 matrix is then searched and the minimum feasible value is saved [3:171]. Finally, the receiver's initial fuel is decremented by 20,000 pounds and the entire process repeated until the minimum fuel solution for both rendezvous point and initial fuel is determined [3:171-172]. It is important to note that this enumeration technique has a coarse grid and requires adjustment when applied to different scenarios.

The stochastic model was a simulation written in SLAM and it was used to verify the operational feasibility of the results obtained from FLTPLN. It did so by taking the "flight plans" produced by the analytic model and "flying them" numerous times with simulated delays and wind variations [3:179].

Bordelon and Marcotte intended to verify the two previously mentioned hypotheses however, the results did not support them [3:64-65]. Instead they found the following:

> 1. The optimal rendezvous can only be determined by analyzing the interaction of the airlifter route distance, the cargo load, and the location of the tanker base [3:x].
> 2. The relative efficiencies of the two aircraft to carry fuel to the rendezvous point... are the determining considerations for optimal takeoff fuel loads [3:57].

3. The optimal takeoff fuel loads are dependent on the aircraft combination and will result in the smallest sum of the total fuel-carrying capacity used [3:xi].

Bordelon and Marcotte were unable to find a heuristic that would solve the problem of rendezvous point location. Instead, their finding that the interactions of several factors determined this location, tends to cast doubt on the value of heuristics used to find "optimal" rendezvous points. If efficiency is the goal, then their findings concerning the initial fuels also cast doubt on the current method of determining the airlifter's initial fuel. Further investigation into the merits of current flight planning techniques are presented in Chapters 4 and 5. Overall, the findings of Bordelon and Marcotte tend to support the idea that the air refueling problem is a candidate for solution by mathematical programming.

2.3.2 Yamani and Coffman. Five years later, Abdulrahman Yamani completed his dissertation, *Analysis of an Air Transportation System.* In this work, Yamani explored ways of formulating math programs for the study of various airlift scenarios. One of them was the problem considered here. He took specific range data and formulated the problem of minimizing mission fuel cost as a nonlinear math program.

The formulation begins with a linear model of the specific range function taken from charted data [26:17]. Next, he derives the various fuel functions and range equations that compose the objective function and constraints. In order to solve the problem, it is broken down into a main problem of finding the rendezvous point and a subproblem of finding the initial fuel [26:47-55]. The subproblem is reduced to one variable because the tanker's initial fuel is dependent on the initial fuel of the receiver [26:47]. Yamani states that the subproblem is convex and therefore provides a unique solution when solved with a line search technique [26:48]. The main problem is rewritten in an equivalent form in order to show that the objective function is convex [26:51-52]. Since the feasible region of the main problem is a convex set, the convergence of Yamani's formulation to a unique optimal solution is guaranteed [26:50,55-56]. This is perhaps the most important result of Yamani's work on the AR flight planning problem. The fact that the problem can be modeled as a convex function on a convex feasible region means there exists a unique combination of the rendezvous location and the initial fuel loads for each mission scenario that minimizes the total fuel cost.

In 1984, Charles Coffman completed his master's thesis *Finding Optimal Fuel and Mid-Air Refueling Location Requirements for C-5A Aircraft.* Coffman's research centered on the development of BASIC code to solve a formulation identical to the one presented in Yamani's dissertation [7]. The solution method that Coffman applied is also identical to the one presented by Yamani [7]. Coffman includes Yamani as a reference indicating the source to be Yamani's then-unpublished dissertation [7:37]. However, Coffman uses no form of in-line documentation in his thesis thereby making it unclear just how much of the formulation and solution method was his work and how much of it was Yamani's. After reading the works of both authors, it seems quite certain that the formulation is truly Yamani's work and that Coffman's thesis does not adhere to a strict documentation standard. In his research, Coffman examined five mission scenarios and numerical results for rendezvous location and initial fuel were reported for each [7:36]. Interestingly, the code took less than five minutes to run on a Commodore 64 microcomputer [7:36]. These numerical results were verified by Yamani, who ran similar, if not identical code on a Vax 11/750 [26:58]. The numerical results obtained by Yamani agree almost exactly with those found by Coffman, however the Vax 11/750 was able to reduce the computation time to about one second and Yamani does not cite Coffman as a source [26:59].

2.4 Non-Linear Optimization

A non-linear program is an optimization problem in which changes in the decision variables do not produce proportional changes in the objective function, or the effects of one or more decision variables are influenced by the values of others, or both [25:52]. Because of this, the well-known and reliable solution techniques used for linear programming problems do not apply to NLP [25:52]. Fortunately, there are a number of solution methods available for NLP's but some are much better at certain types of problems than others.

The most general type of NLP is one with a non-linear objective function and a feasible region defined by non-linear equality and inequality constraints. These types of problems can be solved by a variety of methods including: Sequential Quadratic Programming (SQP), the Generalized Reduced Gradient (GRG) method, the Method of Multipliers (MOM), or various penalty function methods [20:548]. Because of the availability of proven software packages that apply these methods, it is not necessary to write unique code to handle a particular problem. For example, the (Generalized Interactive Non-linear Optimizer) GINO software package applies the GRG method, and can be used to solve most NLPs [18:142]. It is very easy to use and accepts input similar to how the problem appears on paper. MINOS is another GRG-based code [20:555]. It may be called in a FORTRAN program or utilized by a user-friendly "front end" package such as GAMS. Examples of MOM and penalty method codes include BIAS and SUMT respectively [20:555]. Of these, GINO is perhaps the easiest to use. According to Schittkowski, the best methods available for solving NLPs that contain fewer than 100 variables and differentiable problem functions are those that rely on Sequential Quadratic Programming [21]. A detailed description of Schittkowski's code NLPQLD, an implementation of SQP, is given in Appendix 5.3.2.

III. Model Development

3.1 Overview

The problem stated in Chapter 1 was solved and the research objectives were met through the models detailed in this chapter. A total of five models were developed in a process of incremental enhancement so that the last model contains all of the improvements. The most complete model, Model 5 was then modified to solve the related problem of maximizing the cargo load. This evolutionary process tends to keep the models easy to understand and easy to build. Each of the models was solved using sequential quadratic programming and the numerical results are presented in Chapter 4. The reasons for using SQP as the method of solution are outlined in Section 3.3.2 of this chapter.

3.2 Basic Formulation

The first model provides the foundation for all of the others. It is almost a direct implementation of Yamani's work. It finds the optimal refueling point and initial fuel load for an air refueling mission involving a given origin, destination, and tanker base. It does so with the following assumptions:

- The flight characteristics of the tanker and the receiver aircraft are identical and based upon the performance of the C-5A at 31,000 feet
- The aircraft begin and end their mission at altitude over an airbase
- The air refueling occurs instantly at a point in space
- There is no wind
- The aircraft follow a great circle arc from one point to the next.

The first four assumptions are addressed in subsequent models.

Consider an aircraft flying at a constant altitude and airspeed in cruise flight. The four forces; lift, weight, thrust, and drag are exactly balanced. The Gross

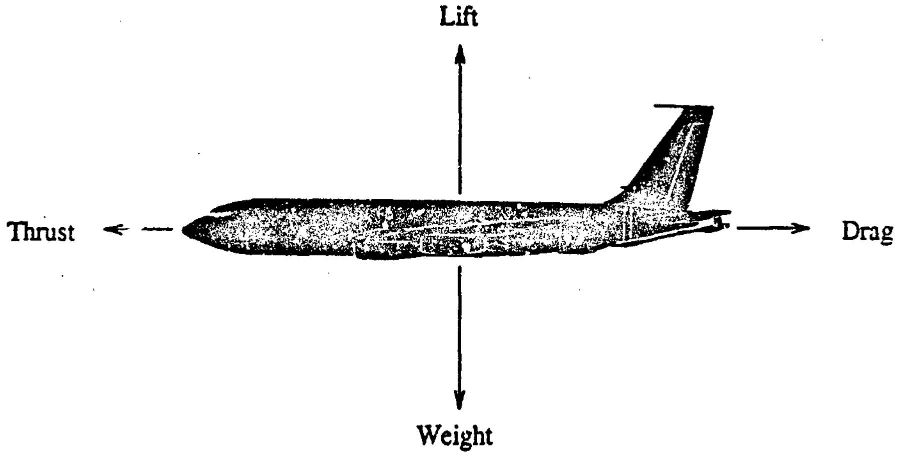

Figure 1. The Four Forces

Weight (GW) of the aircraft is the sum of the aircraft Empty Weight (EW), which is considered to include the aircrew, the payload or cargo weight (w), and the weight of fuel in the tanks (f), expressed in pounds [26:20]:

$$GW = EW + w + f.$$

The direction of this force is toward the center of the earth. For this model, the only item that affects GW is the amount of fuel in the tanks, so gross weight,

$$GW = GW(f),$$

is a function of fuel alone [26:20].

The force of lift,

$$L = \frac{1}{2}\rho V_\infty^2 S C_l(\alpha),$$

at a constant altitude and airspeed is a function of angle of attack, (α), alone [16:550]. In order to fly the aircraft at a constant altitude and airspeed, the pilot trims the aircraft (adjusts the angle of attack) so that exactly enough lift is produced to overcome the current weight. That means the amount of lift required is directly proportional to the gross weight. It can therefore be said that the required angle of attack to maintain the cruise flight condition is a function of GW.

The drag force has two components. These are induced drag,

$$D_i = \frac{C_l(\alpha)^2}{2\pi AR}\rho V_\infty^2 S$$

[16:550], and parasite drag,

$$D_p = \frac{1}{2}\rho V_\infty^2 S C_D$$

[14:425]. Parasite drag is a function of airspeed, air density, and the shape and size of the aircraft. Therefore, parasite drag is constant for a constant altitude and airspeed. Induced drag is an unavoidable by-product of lift. Because induced drag increases with the angle of attack, and the amount of lift necessary varies directly with gross weight, the force of drag is also a function of gross weight.

This model assumes that the thrust produced by the engines acts only to overcome drag. In other words, the vertical component of thrust is negligible. Therefore, the cruise flight condition requires that exactly enough thrust be produced to overcome the drag. Since the amount of drag is a function of gross weight, then the required thrust is also a function of gross weight. In the range of considered flight conditions, it is assumed that thrust is directly proportional to engine fuel flow. Therefore, the engines have to burn more fuel to maintain the cruise flight condition when the aircraft is heavy than when it is light. That means, at the start of a leg, the aircraft is heavier and therefore less fuel efficient than it will be at the end of the leg where it is lighter due to fuel burn off. In short, the fuel efficiency of an aircraft, in terms of Nautical Air Miles (NAM) per 1000 pounds of fuel burned at a constant altitude is some function of airspeed and gross weight. This is reflected in Figure 2, taken from Yamani's article [27:793].

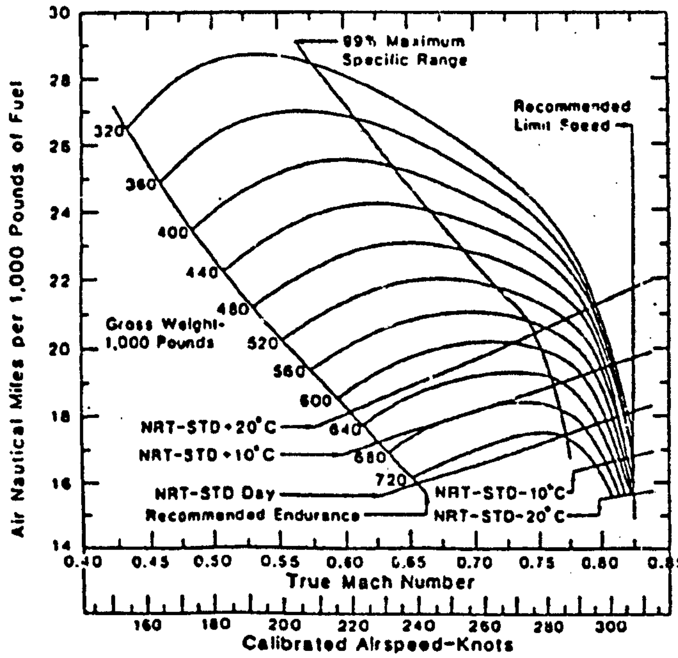

Figure 2. Specific Range Chart for C-5A at 31,000 feet

Notice that, at any given airspeed, NAM decreases with an increase in GW. Due to the presence of parasite drag, it is advantageous to slow down at lighter gross weights. That is why the peak of each curve occurs at different airspeed. In order to get the maximum possible range out of the aircraft, a pilot would fly the aircraft at the airspeeds corresponding to the peak of each curve. However, doing so would mean flying at dangerously slow airspeeds. In order to avoid that problem, a pilot can fly at a faster airspeed for each gross weight and achieve 99% of the maximum possible range. This is reflected by the 99% maximum range line.

One of the assumptions made by Yamani, and also made here, is that the fuel efficiency, NAM, is a linear function of GW [26:20]. That is:

$$NAM = a_0 + a_1(GW)$$

In Yamani's model, the regression coefficients a_0, and a_1 are found by fitting a first order linear model to the data obtained from the 99% maximum range curve. Yamani also shows that, along this curve, a quadratic fit is better [26:20]. This is illustrated in Figure 3, also taken from Yamani's article [27:794].

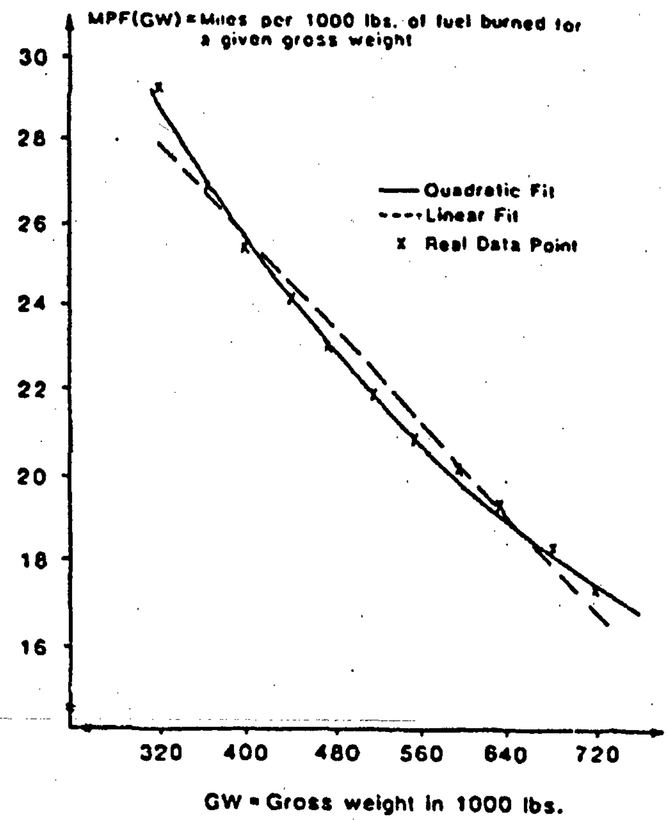

Figure 3. The Linear and Quadratic fit

3.2.1 The Range Equation. This section covers Yamani's development of the range equation from the fuel mileage function. Range is obtained by integrating NAM with respect to GW

$$R = \int_{emptytanks}^{fulltanks} NAM(GW)dGW$$

$$R = \int_{EW+w}^{EW+w+g} a_0 + a_1(GW)dGW.$$

Fuel is the only thing that affects GW so

$$dGW = df,$$

and the integral becomes

$$R = \int_{EW+w}^{EW+w+g} a_0 + a_1(EW + w + f)df.$$

The result is the following range equation [26:21]:

$$R(g, w) = (a_0 + a_1(EW + w + \frac{g}{2}))g.$$

3.2.2 The Fuel-Consumed Function. According to Yamani, the fuel consumed function determines the amount of fuel consumed when an aircraft departs with initial fuel g, and flies a distance d. If the aircraft takes off with a fuel load g, then after flying some distance d, it will have f_r pounds of fuel remaining. Therefore the Fuel Consumed (FC) by flying a distance d is

$$FC = g - f_r.$$

The distance d, can by found by integrating the fuel mileage function with respect to fuel as in the range equation:

$$d = \int_{f_r}^{g} a_0 + a_1(EW + w + f)df$$

$$d = a_0(g - f_r) + a_1(EW + w)(g - f_r) + \frac{a_1}{2}(g^2 - f_r^2).$$

Solving for f_r yields

$$f_r = -\frac{a}{a_1} \pm \frac{\sqrt{(a+a_1g)^2 - 2a_1d}}{a_1}$$

where

$$a = a_0 + a_1(EW + w).$$

Now, only the positive root makes f_r physically possible, and Yamani's fuel consumed function can be stated [26:23,24]:

$$FC(g,d) = g - f_r = g + \frac{a}{a_1} + \frac{\sqrt{(a+a_1g)^2 - 2a_1d}}{a_1}$$

3.2.3 The Fuel-Required Function. Yamani's Fuel-Required function, FR, determines the minimum amount of fuel required by the aircraft to fly a distance d. If d is set equal to the range equation, and solved for the initial fuel g, then the minimum fuel required to fly the distance d can be determined.

$$d = R(g,w) = (a_0 + a_1(EW + w + \frac{g}{2}))g$$

Solving for g gives:

$$g = -w - \frac{a}{a_1} + \frac{\sqrt{(a+a_1g)^2 - 2a_1d}}{a_1},$$

and [26:26,27]:

$$FR(d) = g = -w - \frac{a}{a_1} + \frac{\sqrt{(a+a_1g)^2 - 2a_1d}}{a_1}.$$

3.2.4 The Measurement of Distance on a Spherical Earth. The shortest distance between two points on a sphere is the length of the Great Circle, (GC) arc connecting them [26:103]. When the coordinates of those points are given in latitude

(L) and longitude (λ) then the length of the great circle arc is given by [26:110]:

$$D(L_1, \lambda_1, L_2, \lambda_2) = (R + alt) \arccos(\sin(L_1)\sin(L_2) + \cos(\lambda_2 - \lambda_1)\cos(L_1)\cos(L_2))$$

3.2.5 Summary of functions and notation. At this point, it is worthwhile to assemble the equations and notation. The specific problem this model solves is the selection of the optimal rendezvous point and initial fuels for a mission where a cargo aircraft departs from a destination base and is refueled enroute by a tanker that departs and recovers to a third base. The locations of the bases and the rendezvous point are described by their latitude and longitude coordinates. This is shown in Figure 4 taken from Yamani [26:46]. The following notation is similar to that used by Yamani:

o_{lat} = Latitude of the receiver's origin base

o_{long} = Longitude of the receiver's origin base

d_{lat} = Latitude of the receiver's destination

d_{long} = Longitude of the receiver's destination

t_{lat} = Latitude of the tanker base

t_{long} = Longitude of the tanker base

ϕ = Latitude of the rendezvous point

θ = Longitude of the rendezvous point

g = Initial fuel load of the receiver aircraft

h = Initial fuel load of the tanker aircraft

EW_c = Empty weight of the receiver (cargo) aircraft

EW_t = Empty weight of the tanker aircraft

w = Cargo weight

$MaxTO_c$ = Maximum take off weight of the cargo aircraft

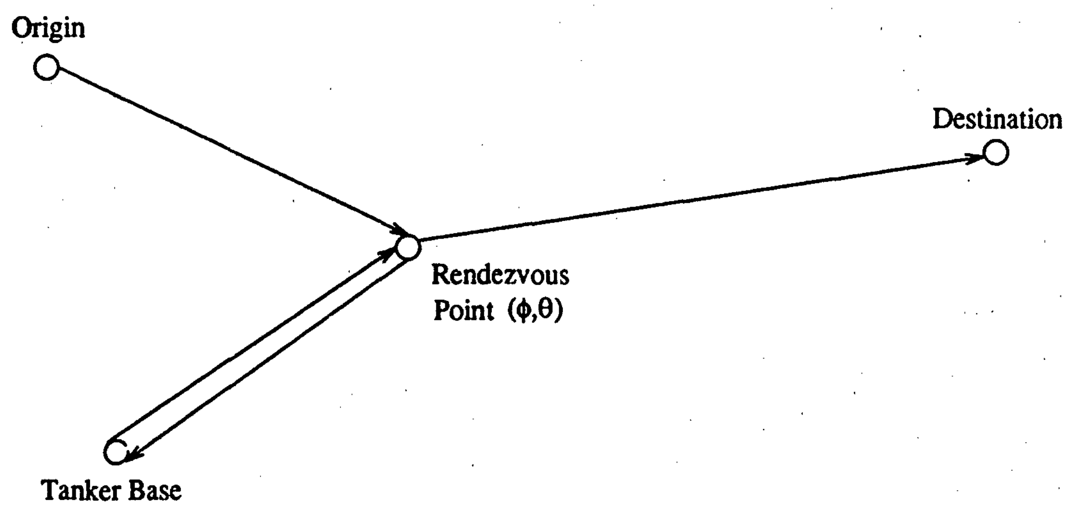

Figure 4. The Relative Locations

$MaxTO_t$ = Maximum take off weight of the tanker aircraft

$R = 3443.92$, The mean radius of the Earth in nautical miles [2:429]

$1NM = 6076.12$ Feet [2:429]

$alt = 31{,}000$ feet or 5.102 nautical miles

The following equations:

$$D_{or}(\phi,\theta) = (R + alt)\arccos(\sin(o_{lat})\sin(\phi) + \cos(\theta - o_{long})\cos(o_{lat})\cos(\phi)),$$

$$D_{rd}(\phi,\theta) = (R + alt)\arccos(\sin(d_{lat})\sin(\phi) + \cos(d_{long} - \theta)\cos(d_{lat})\cos(\phi))$$

and,

$$D_{br}(\phi,\theta) = (R + alt)\arccos(\sin(t_{lat})\sin(\phi) + \cos(\theta - t_{long})\cos(t_{lat})\cos(\phi))$$

define the distance from the origin to the rendezvous point, the distance from the rendezvous point to the destination, and the distance from the tanker base to the rendezvous point respectively [26:41,42]. Since the AR is considered to take place instantaneously at a point in space, the distance from the rendezvous point to the tanker base is equal to the distance of the reverse [26:15]. In Yamani's formulation and in this first model, it is assumed that the regression coefficients are identical for the receiver and the tanker aircraft. In other words, they have the same fuel mileage function [26:50].

Yamani defines the following set of fuel functions:

$$\begin{aligned}
FCc(g,\phi,\theta) &= g + \frac{a_0 + a_1(EW_c + w)}{a_1} \\
&+ \frac{\sqrt{(a_0 + a_1(EW_c + w + g))^2 - 2a_1 D_{or}(\phi,\theta)}}{a_1},
\end{aligned}$$

$$\begin{aligned}
FRc(\phi,\theta) &= -w - \frac{a_0 + a_1(EW_c + w)}{a_1} \\
&+ \frac{\sqrt{(a_0 + a_1(EW_c + w + g))^2 - 2a_1 D_{rd}(\phi,\theta)}}{a_1},
\end{aligned}$$

$$\begin{aligned}
FCt(h,\phi,\theta) &= h + \frac{a_0 + a_1 EW_t}{a_1} \\
&+ \frac{\sqrt{(a_0 + a_1(EW_t + h))^2 - 2a_1 D_{br}(\phi,\theta)}}{a_1}
\end{aligned}$$

and,

$$\begin{aligned}
FRt(\phi,\theta) &= -\frac{a_0 + a_1 EW_t}{a_1} \\
&+ \frac{\sqrt{(a_0 + a_1(EW_t + h))^2 - 2a_1 D_{br}(\phi,\theta)}}{a_1}
\end{aligned}$$

respectively as the fuel consumed by the receiver to reach the rendezvous point, the fuel required by the receiver aircraft to reach the destination, the fuel consumed by the tanker to reach the rendezvous point, and the fuel required by the tanker for its return to base [26:50].

3.2.6 The Objective Function. The objective of this NLP is to minimize the total fuel cost of the mission. Therefore the objective function is the sum of the fuel functions for both the tanker and the receiver aircraft [26:44]

$$V(g,h,\phi,\theta) = FCc(g,\phi,\theta) + FRc(\phi,\theta) + FCt(h,\phi,\theta) + FRt(\phi,\theta).$$

3.2.7 The Feasible Region. It can be seen from the objective function that there are four decision variables: ϕ, θ, g, and h. The purpose of this section is to explain the constraints that define the feasible region. It is easy to conceptualize this problem as a superposition of a fuel problem in g and h onto a spatial problem in θ and ϕ. Each of the initial fuel variables, g and h, must be at least large enough to bring the rendezvous point within range of their respective aircraft. They also must be less than some value determined by the maximum takeoff weight of the aircraft [26:45]. For the purpose of this model, the maximum initial fuel for the cargo aircraft is a function of cargo weight and the maximum initial fuel for the tanker aircraft is a constant number [26:14]. That is,

$$g_{max}(w) = MaxTO_c - EW_c - w$$

and

$$h_{max} = MaxTO_t - EW_t.$$

The fuel required for the cargo aircraft to reach the rendezvous point is [26:45]

$$FRca(\phi,\theta) = -w - \frac{a_0 + a_1(EW_c + w)}{a_1} + \frac{\sqrt{(a_0 + a_1(EW_c + w + g))^2 - 2a_1 D_{or}(\phi,\theta)}}{a_1}.$$

This is simply the fuel required function with the distance $D_{or}(\phi,\theta)$ instead of $D_{rd}(\phi,\theta)$. The minimum fuel for the tanker aircraft to make a round trip to the rendezvous point is [26:45]

$$FRta(\phi,\theta) = -\frac{a_0 + a_1 EW_t}{a_1} + \frac{\sqrt{(a_0 + a_1(EW_t + h))^2 - 4a_1 D_{br}(\phi,\theta)}}{a_1}.$$

This is the fuel required function modified by replacing $D_{or}(\phi,\theta)$ with $2D_{or}(\phi,\theta)$. These values bound g and h for the problem and give the following constraints [26:45]:

$$\begin{aligned} g &\geq FRca(\phi,\theta) \\ g &\leq g_{max}(w) \\ h &\geq FRta(\phi,\theta) \\ h &\leq h_{max} \end{aligned}$$

The decision variables, ϕ and θ, define the location of the rendezvous point. If g and h are set to their maximum values, the objective function can be calculated as a surface above the plane formed by θ and ϕ. The following three constraints define the region that must contain all of the feasible rendezvous points [26:45].

- The distance from the origin to the refueling point cannot exceed the maximum range of the receiver [26:45]:

$$D_{or}(\phi,\theta) \leq R_c(g_{max}(w), w).$$

- The distance from the refueling point to the destination cannot exceed the maximum range of the receiver [26:45]:

$$D_{rd}(\phi,\theta) \leq R_c(g_{max}(w), w).$$

- The out-and-back distance from the origin to the refueling point cannot exceed the range of the tanker [26:45]:

$$D_{br}(\phi,\theta) \leq \frac{R_t(h_{max})}{2}.$$

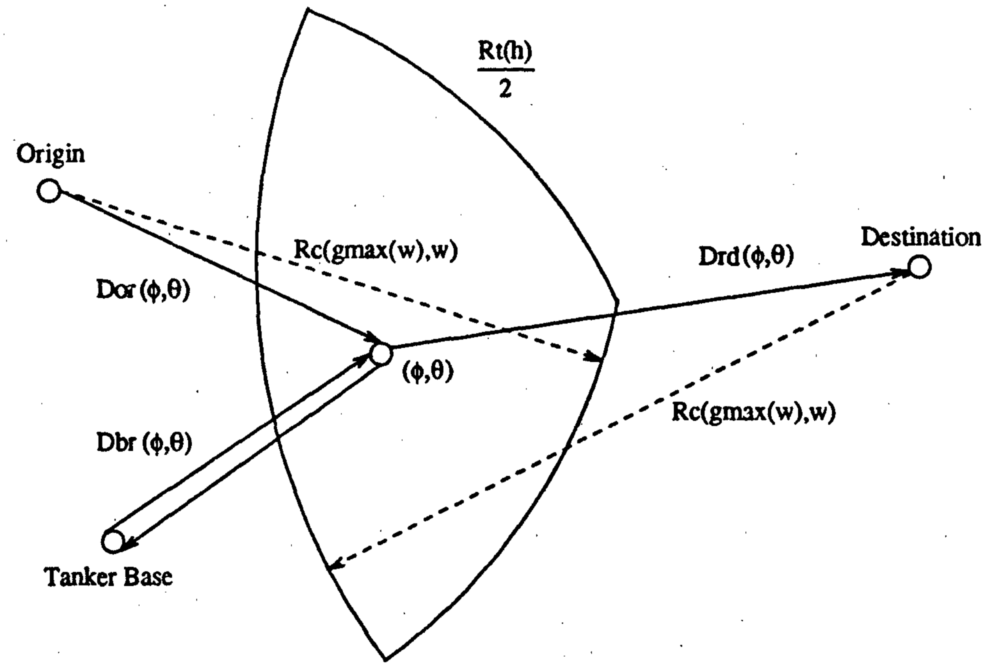

Figure 5. The region in which all feasible rendezvous points must lie

It can now be stated that all feasible points must lie within the region shown in Figure 5 [26:46]. However, not all points within this region are feasible. It is not enough that both aircraft can arrive at the rendezvous point. The tanker must also have sufficient excess fuel for transfer to the receiver aircraft so that it may safely complete its mission [26:45]. To develop this as a constraint, consider the most distant refueling point possible. At this point, both the tanker and receiver aircraft must take off with their tanks full. At the refueling point the tanker has

$$h_{max} - FCt(h_{max}, \phi, \theta)$$

pounds of fuel left. That means that it can afford to give

$$h_{max} - FCt(h_{max}, \phi, \theta) - FRt(\phi, \theta)$$

pounds of fuel to the receiver aircraft. Meanwhile, the receiver aircraft has

$$g_{max}(w) - FCc(g_{max}(w), \theta.\phi)$$

pounds of fuel left at the refueling point. And it needs

$$FRc(\phi, \theta)$$

pounds of fuel to complete its mission. In order for this refueling point to be feasible, the fuel required by receiver must be less than the sum of the fuel left in the receiver and fuel available in tanker. Mathematically:

$$\begin{aligned} FRc(\phi,\theta) &< g_{max}(w) + h_{max} - FCc(g_{max}(w), \phi, \theta) \\ &\quad - FCt(h_{max}, \phi, \theta) - FRt(\phi, \theta). \end{aligned}$$

or [26:47]:

$$\begin{aligned} g_{max}(w) + h_{max} &\geq FCc(g_{max}(w), \phi, \theta) + FCt(h_{max}, \phi, \theta) \\ &\quad + FRc(\phi, \theta) + FRt(\phi, \theta). \end{aligned}$$

The equality holds at the most costly feasible refueling point.

Any given refueling point only calls for enough initial fuel, g and h, to make the mission feasible. If more than the required amount is carried then the total fuel cost increases because of the corresponding increase in gross weight [26:45]. At optimality, the following constraint must always be binding [26:47]:

$$g + h = FCc(g, \phi, \theta) + FCt(h, \phi, \theta) + FRc(\phi, \theta) + FRt(\phi, \theta)$$

Therefore, the final constraint is defined by [26:47]:

$$g + h \geq FCc(g, \phi, \theta) + FCt(h, \phi, \theta) + FRc(\phi, \theta) + FRt(\phi, \theta)$$

The following is a summary of the objective function and the constraints that define Model 1.

$$\min V(g, h, \phi, \theta) = FCc(g, \phi, \theta) + FRc(\phi, \theta) + FCt(h, \phi, \theta) + FRt(\phi, \theta)$$

Subject to:

$$\begin{aligned}
D_{or}(\phi, \theta) &\leq R_c(g_{max}(w), w) && (1)\\
D_{rd}(\phi, \theta) &\leq R_c(g_{max}(w), w) && (2)\\
D_{br}(\phi, \theta) &\leq \frac{R_t(h_{max})}{2} && (3)\\
g_{max}(w) + h_{max} &\geq FCc(g_{max}(w), \phi, \theta) + FCt(h_{max}, \phi, \theta) && \\
&\quad + FRc(\phi, \theta) + FRt(\phi, \theta) && (4)\\
g + h &\geq FCc(g, \phi, \theta) + FCt(h, \phi, \theta) + FRc(\phi, \theta) + FRt(\phi, \theta) && (5)\\
g &\geq FRca(\phi, \theta) && (6)\\
h &\geq FRta(\phi, \theta) && (7)\\
g &\leq g_{max}(w) && (8)\\
h &\leq h_{max} && (9)
\end{aligned}$$

The entire formulation of Model 1 is attributable to Yamani. This formulation has been solved by both Coffman and Yamani using decomposition and search methods.

3.3 Solution Methods

3.3.1 Yamani's Solution. Yamani begins by rewriting the objective function and proving it to be convex [26:53,54]. That means the objective function, as written here, is also convex. Next, he decomposes the problem into a main problem in θ and ϕ and a subproblem in g. The solution space for the main problem and the fuel subproblem are shown to be convex sets [26:53,54]. This is done to prove that the local minimum found by the solution procedure is, in fact, be the optimum solution.

The variable h is actually dependent on g so it can be eliminated to reduce the dimensionality of the problem [27:796]. He initializes the problem by setting $g = g_{max}(w)$ and picking an interior point for θ and ϕ. Next, he finds the minimum of the spatial problem using a search algorithm. The result is transferred to the fuel problem which is minimized by way of a line search in g. This in turn is fed back into the main problem and the process is repeated until little change is noted between iterations [27:797-99]. This decomposition and search strategy is effective, however better solution methods are available. The interested reader should refer to [26] for more information on Yamani's solution technique.

3.3.2 Solution by Sequential Quadratic Programming. For this research, the problem formulation is not modified to facilitate solution. Rather, the formulation is solved "as-is" by Sequential Quadratic Programming. The reasons for this choice are twofold. First, the nature of Yamani's formulation and the lack of an "arccosine" function precluded the use of both GINO and GAMS and second, Dr. Schittkowski himself was available to provide and explain the SQP code. According to Dr. Schittkowski, SQP as a method is the best algorithm available for solving constrained NLP's under the following conditions [21]:

- The problem is smooth. That is, the objective function must be differentiable at least twice and the constraints at least once

- The problem is not too large
- The problem is fairly well scaled
- The problem is well defined

This problem fits the above criteria and therefore SQP is an appropriate means for solving this NLP and obtaining numerical results. Because the explanation of SSQP is lengthy, the details are covered in Appendix 5.3.2.

For models 1 and 2, the objective function is convex and the feasible region is a convex set. However, this becomes difficult to show for subsequent models. Graphs of the feasible region suggest convexity, but they are inconclusive and not available for all of the models. Therefore, the solutions obtained do not meet the strict mathematical definition of optimality. The feasibility of each numerical solution is demonstrated in the program output and the reasonability of the solutions can be readily shown. Also, the solutions obtained for Model 1, Yamani's formulation, are quite close to his published results, and each model always gives the same solution when started from different initial points. For these reasons, great confidence is placed in the SQP code and the solutions it finds to this problem.

3.4 Model 2

Now that Yamani's formulation is understood and readily solved it is time to begin the enhancements. These include the incorporation of KC-135E and C-141B performance data as well as a different flight profile.

Aircraft performance manuals were obtained for the KC-135E and the C-141B. Figure 6 shows the specific range chart for the KC-135E at 31,000 feet. Notice that it is not possible to fly the 99% maximum range line at a constant airspeed. In order to achieve 99% max range the pilot would have to be constantly reducing the airspeed. If this is done, then the job of flying the aircraft can become much more difficult. It would be even worse for the air traffic controller who would not

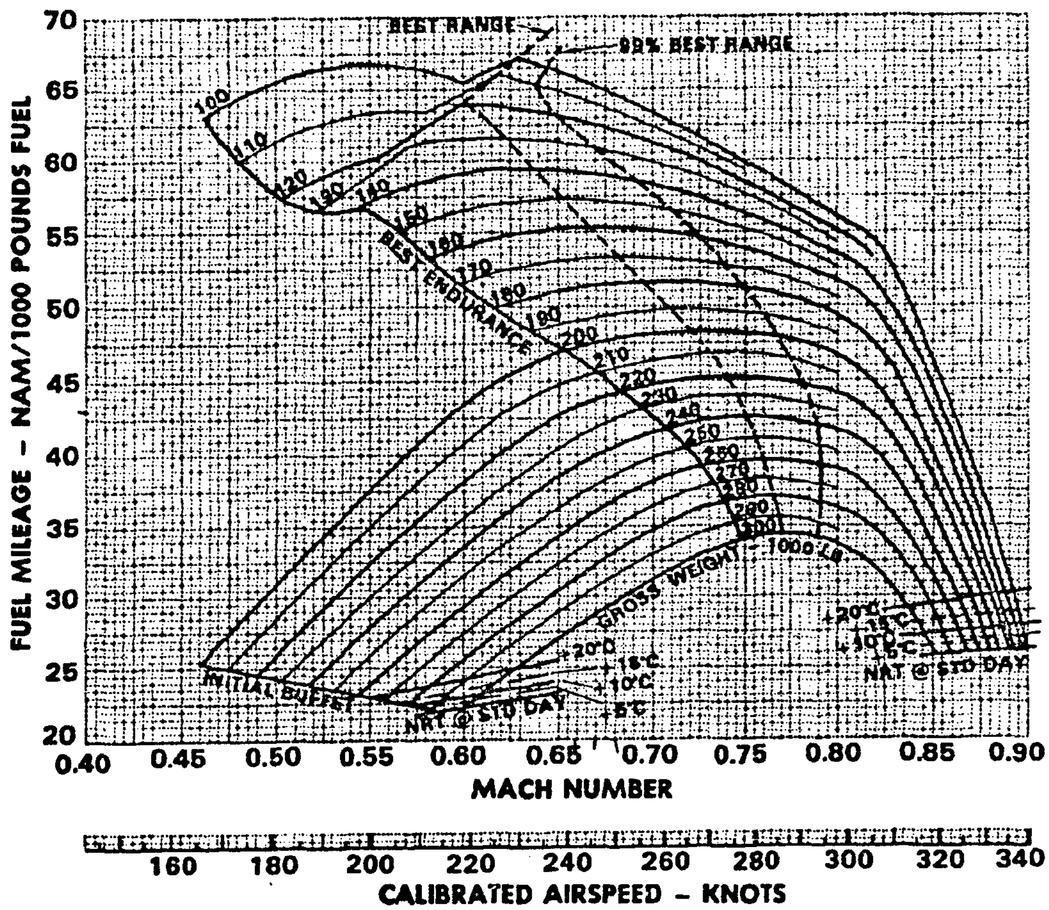

Figure 6. Specific range chart for the KC-135E at 31,000 feet

be able to predict the future location of an aircraft flown in this manner. Other airplanes following at a constant speed would catch up and present a traffic conflict. Finally, AR requires that both aircraft reach the designated rendezvous point at the same time. Flying at a constantly changing airspeed would make this job extremely difficult. For these reasons, the missions are flown at a constant airspeed. For the KC-135E that airspeed is 0.75 mach [6]; the C-141B cruises at 0.74 mach [5].

The next step is to install the tanker and transport data in Model 1. This begins with the coefficients of the $NAM(GW)$ function for each aircraft. They are determined in almost the same manner as Yamani's formulation. A vertical line is drawn at the appropriate mach number and the intersections of the gross weight curves with this line become the data points. Linear regression is used in the same manner as Model 1, to estimate the slope and intercept coefficients for $NAM(GW)$. Figure 7 is a graph of the data and the straight-line fit. Notice that the fit happens

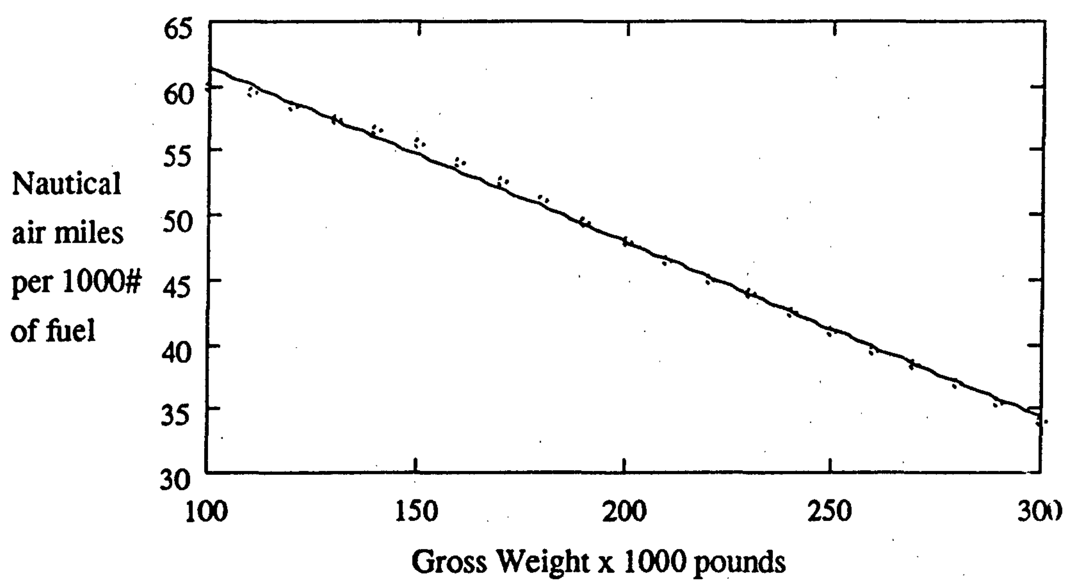

Figure 7. Linear fit to KC-135E specific range data

to be quite good. The coefficient of correlation of -0.9983 is equal to the one reported by Yamani for the quadratic fit to his NAM(GW) function [26:20]. The process was repeated for the C-141B data and the regression coefficients are given in Table 2. The coefficient of correlation is -0.99685, meaning this fit is quite accurate as well.

In Model 1, the NAM functions for the tanker and the receiver were assumed to be identical, so the notation of $a0$ and $a1$ was used throughout. For this model onward, $b0$ and $b1$ denote the NAM coefficients for the tanker aircraft. Still, a few more constants are needed to integrate the new aircraft into the model. The C-141 weight data comes from section 5 of the aircraft manual [8:5-2], whereas the KC-135 data is estimated. Table 1 summarizes the aircraft weight data and Table 2 gives the regression coefficients used in Model 2.

Table 1. Aircraft weight data, in units of 1000 pounds

	C-141B	KC-135E
Max Takeoff Weight	323.100	300.00
Empty Weight	152.685	100.00
Max Fuel Load	151.452	200.00

Table 2. Regression Coefficients for the fuel mileage functions

	C-141B	KC-135E
$a0$	45.9127	--
$a1$	-0.0531	--
$b0$	--	74.9700
$b1$	--	-0.1353

To complete Model 2, the next order of business is the mission profile. This new profile has the C-141 departing from Aviano, Italy for McGuire AFB, New Jersey. It will be refueled enroute by a KC-135 based at the Azores Islands. This profile was chosen for comparison to a computer-generated flight plan for what is essentially the same mission (This flight profile is used for the remaining models as well). The lat-long coordinates given in Table 3 for Aviano and McGuire come from the computer-generated flight plan. The coordinates of Lajes, Azores Is. were found in the Flight Information Handbook [13:C-15].

Table 3. Airbase Latitude and Longitude coordinates

		Latitude	Longitude
Departure base	Aviano Italy	46.03 N	12.6 E
Destination	McGuire AFB NJ	40.02 N	74.6 W
Tanker base	Azores	38.70 N	27.1 W

Simply replacing the appropriate numbers in Model 1 gives Yamani's model with the KC-135E and C-141B aircraft as well as the new flight profile. Numerical results for this model are reported in Chapter 4.

3.5 Model 3

While Model 2 represents an implementation of Yamani's model, there is still a long way to go. This third model explores the first actual changes to Yamani's formulation. More specifically, Model 3 contains the following modifications to Model 2:

- Takeoff and climb, as well as descent and landing, are assumed to require 100 nautical air miles to complete.
- A fuel cost, proportional to initial gross weight, is included to account for takeoff and climb to altitude.
- Descent, and Pattern, Approach and Landing (PAL) have a fixed fuel cost.
- The aircraft must always land with a fixed reserve fuel.
- The cargo aircraft must be able to divert to an alternate from the rendezvous point.

This model assumes a minimum flight distance of 100 nautical miles in order to account for realistic climbout and descent. The actual distance required for climbout is a function of several factors including wind, temperature, and gross weight. However, examination of the charts in the performance manual suggests that one hundred nautical miles is a likely average for the C-141B to climb to 31,000 feet at typical gross weights [9:4-8]. The same thing can be said about the KC-135E [10:1A4-14]. The reason to assume the aircraft begins descent 100 nautical miles from their destination is the common use of long, straight-in approaches. Such an approach utilizes "vectors to final" instead of a published penetration descent.

Consider the case of a refueling point occurring directly overhead of the tanker base. The calculated distance from the tanker base to the rendezvous point would be zero, or: $D_{br}(\phi, \theta) = 0$. When this occurs, the method of calculating fuel consumption in previous models would result in a zero fuel cost for the tanker. That is, $FCt(h, \phi, \theta) = 0$ and $FRt(\phi, \theta) = 0$ when $\phi = t_{lat}$ and $\theta = t_{long}$. Now, because of the conditional statement, the distance is set to 100 nautical miles and the entire distance is spent in a large climbing turn. For the return, the 100 nm may be thought of as a large descending turn. The updated methods of fuel computation are discussed next.

In order to model the climb fuel costs, this model makes use of data taken from the "additional fuel required to climb" charts in the performance manuals. These chart allow the flight planner to determine how much more fuel would be required to climb from sea level to a given altitude than would be needed to cruise at that altitude for a distance equal to the climb distance. In other words, if it takes 100 nautical miles to climb to 31,000 feet then the total fuel cost of this climb is equal to the fuel required to cruise for 100 nautical miles at 31,000 feet plus the additional fuel required to climb to the same altitude [10:1A4-4]. This fits very nicely into the model because the Yamani formulation starts the aircraft off at altitude over their departure bases. Therefore, in order to account for the climb fuel, the fuel consumed function is increased by a function of the initial gross weight. That is,

$$FCc(g, \phi, \theta) = FCc(g, \phi, \theta) + climb(GW).$$

The function $climb(GW)$ is developed using data from the additional fuel required to climb chart. The chart is shown in Figure 8 for the KC-135E [9:4-15]. A linear fit to the data gives the following:

$$climb(gw) \;=\; c_0 + c_1 GW$$

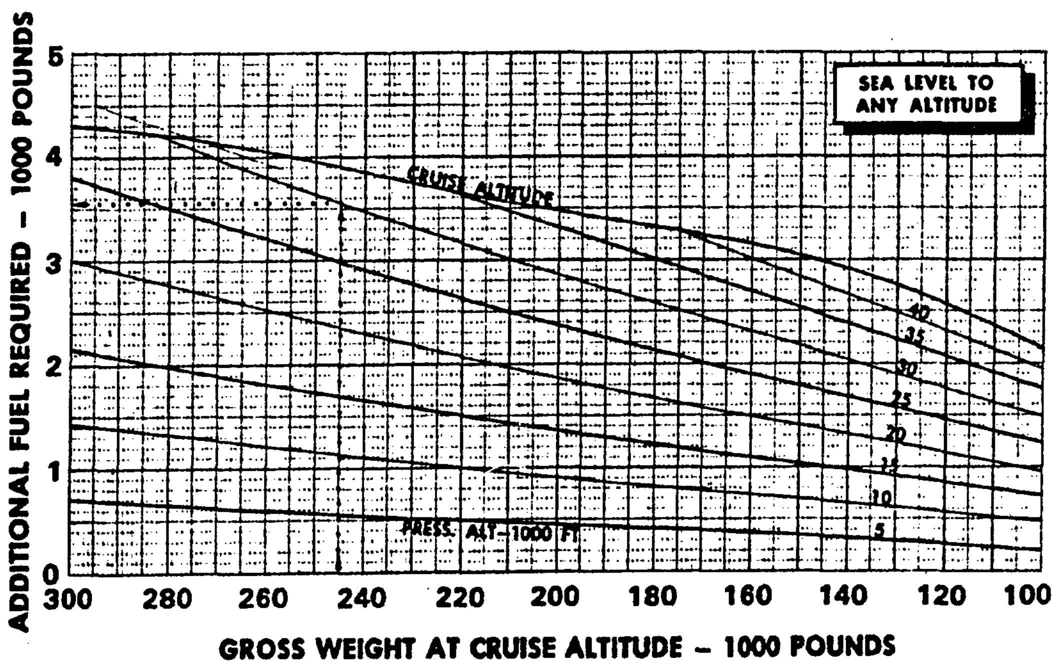

Figure 8. Additional fuel required to climb

where

$$GW = EW_c + w + g$$

The same thing is done for the C-141B and the results are reported in Table 4.

Table 4. Coefficients for Additional Fuel Required to Climb Calculations

	C-141B	KC-135E
c_0	-0.08	--
c_1	0.015	--
d_0	--	-0.127
d_1	--	0.016

The fuel consumed functions are updated to become:

$$FCc(g,\phi,\theta) = FCc(g,\phi,\theta) + c_0 + c_1(EW_c + w + g)$$

and

$$FCt(h,\phi,\theta) = FCt(h,\phi,\theta) + d_0 + d_1(EW_t + h).$$

On the arrival end of the problem, the descent and PAL are assumed to require a fixed amount of fuel. The reserve fuel is calculated based upon AFR 60-16 requirements. The model assumes that a weather alternate is not necessary for either the tanker or the receiver. In this case, only the enroute reserve reouirement applies. For turbojet aircraft, AFR 60-16 requires:

> ...aircraft must carry enough useable fuel on each flight to increase the total planned flight time between refueling points by 10 percent or 20 minutes, whichever is greater. To compute these fuel reserves... use fuel consumption rates that provide maximum endurance at 10,000 feet [11:6].

This results in a reserve requirement of about 40 minutes for the C-141B and 20 minutes for the KC-135E under the assumed scenario.

The descent fuel for each aircraft was estimated from inspection of the descent fuel charts in the aircraft performance manual. For the C-141B, the model uses 1,200 pounds as the descent fuel [9:7-4], 1,300 pounds as the *PAL* fuel [3:19], and 6,700 pounds of fuel as the reserve [9:6-3]. Likewise for the KC-135E, the descent fuel is assumed to be 1,200 pounds [10:A8-3], the *PAL* fuel 1,000 pounds [10:A11-7], and the reserve of 20 minutes at 10,000 feet works out to be approximately 3,000 pounds [10:A6-8]. Table 5 summarizes these fuel costs. Therefore the fuel required function for the airlifter becomes

$$FRc(\phi,\theta) = FRc(\phi,\theta) + desc_c + PAL_c + res_c$$

Table 5. Summary of fuel costs, in units of 1000 pounds

		C-141B	KC-135E
Descent:	*desc*	1.2	1.2
PAL:	*PAL*	1.3	1.0
Reserve:	*res*	6.7	3.0

and, likewise, for the tanker

$$FRt(\phi,\theta) = FRt(\phi,\theta) + desc_t + PAL_t + res_t.$$

Recall that the minimum flight distance is now 100 nautical miles. This is done to make the calculation of climb fuel work properly with the "additional fuel required to climb technique." However, the descent fuel is assumed to be a constant number and the descent distance is assumed to be 100 nautical miles. Therefore, the legs that end at a base, including $Drd(\phi,\theta)$, $Dbr(\phi,\theta)$ when used in $FRt(\phi,\theta)$, and the soon-to-be-defined $Dra(\phi,\theta)$ must all be reduced by 100 nautical miles to prevent a cruise fuel calculation over the descent distance.

Perhaps the most important improvement for Model 3 is the inclusion of a divert requirement for the receiver aircraft. This is often a binding constraint for the mission planner. As such, this constraint is necessary to make the model believable. In order to develop this constraint, a new quantity is required. From Yamani, the weight of fuel left in the tanks of the receiver aircraft at the refueling point is [26:51]:

$$WFL(g,\phi,\theta) = g - FCc(g,\phi,\theta)$$

This amount of fuel must be enough to get the cargo aircraft to its refueling alternate with no additional fuel from the tanker. This imitates the very real concern that the AR may not go as planned due to an accident or bad weather in the area. To handle this, another distance and another fuel required function must be developed.

The distance from the refueling point to the alternate is

$$D_{ra}(\phi,\theta) = (R + alt)\arccos(\sin(a_{lat})\sin(\phi) + \cos(a_{long} - \theta)\cos(a_{lat})\cos(\phi)) - 100.$$

The AR alternate in the computer-generated flight plan is Aviano, which is also the departure base. Therefore, Aviano is the alternate in the model as well. As a result, the coordinates of the AR divert base (a_{lat}, a_{long}) are set equal to those of the origin, (o_{lat}, o_{long}). The fuel required to divert to the alternate and land with reserves is

$$\begin{aligned} FRd(\phi,\theta) &= -w - \frac{a_0 + a_1(EW_c + w)}{a_1} \\ &\quad + \frac{\sqrt{(a_0 + a_1(EW_c + w + g))^2 - 2a_1 D_{ra}(\phi,\theta)}}{a_1} \\ &\quad + desc_c + PAL_c + res_c \end{aligned}$$

Therefore the constraint to be added to the problem becomes:

$$WFL(g,\phi,\theta) \geq FRd(\phi,\theta)$$

Since all of the fuel equations have been modified, the constraints written in terms of distance and range are no longer sufficient. The constraint set must be updated to reflect this. The first constraint

$$D_{or}(\phi,\theta) \leq R_c(g_{max}(w), w)$$

now becomes

$$FCc(g,\phi,\theta) \leq g_{max}(w)$$

in order to account for the fact that it takes more than just cruise fuel to reach the rendezvous point. Likewise, the constraint that keeps the rendezvous point within range of the destination,

$$D_{rd}(\phi,\theta) \leq R_c(g_{max}(w), w)$$

becomes

$$FRc(\phi,\theta) \leq g_{max}(w).$$

The same thing applies to the tanker. The old constraint that made sure the tanker could make a round trip to the rendezvous point,

$$D_{br}(\phi,\theta) \leq \frac{R_t(h_{max})}{2}$$

is now stated as

$$(FCt(h,\phi,\theta) + FRt(\phi,\theta)) \leq h_{max}.$$

Although it may still be considered a constraint for conceptual purposes, $h \leq h_{max}$ was rewritten in the model code as a variable bound. The divert fuel constraint, $WFL(g,\phi,\theta) \geq FRd(\phi,\theta)$ takes its place.

The changes that constitute Model 3 can be summarized by restating the model formulation starting with the objective function.

$$\min V(g,h,\phi,\theta) = FCc(g,\phi,\theta) + FRc(\phi,\theta) + FCt(h,\phi,\theta) + FRt(\phi,\theta)$$

Where:

$$\begin{aligned} FCc(g,\phi,\theta) &= g + \frac{a_0 + a_1(EW_c + w)}{a_1} \\ &+ \frac{\sqrt{(a_0 + a_1(EW_c + w + g))^2 - 2a_1 D_{or}(\phi,\theta)}}{a_1} \\ &+ c_0 + c_1(EW_c + w + g) \end{aligned}$$

$$FCt(h,\phi,\theta) = h + \frac{a_0 + a_1 EW_t}{a_1} + \frac{\sqrt{(a_0 + a_1(EW_t + h))^2 - 2a_1 D_{br}(\phi,\theta)}}{a_1} + d_0 + d_1(EW_t + h)$$

$$FRc(\phi,\theta) = -w - \frac{a_0 + a_1(EW_c + w)}{a_1} + \frac{\sqrt{(a_0 + a_1(EW_c + w + g))^2 - 2a_1 D_{rd}(\phi,\theta)}}{a_1} + desc_c + PAL_c + res_c$$

$$FRt(\phi,\theta) = -\frac{a_0 + a_1 EW_t}{a_1} + \frac{\sqrt{(a_0 + a_1(EW_t + h))^2 - 2a_1 D_{br}(\phi,\theta)}}{a_1} + desc_t + PAL_t + res_t$$

$$D_{or}(\phi,\theta) = (R + alt)\arccos(\sin(o_{lat})\sin(\phi) + \cos(\theta - o_{long})\cos(o_{lat})\cos(\phi))$$

$$D_{br}(\phi,\theta) = (R + alt)\arccos(\sin(t_{lat})\sin(\phi) + \cos(\theta - t_{long})\cos(t_{lat})\cos(\phi))$$

When used in $FRt(\phi,\theta)$

$$D_{br}(\phi,\theta) = (R + alt)\arccos(\sin(t_{lat})\sin(\phi) + \cos(\theta - t_{long})\cos(t_{lat})\cos(\phi)) - 100$$

$$D_{rd}(\phi,\theta) = (R + alt)\arccos(\sin(d_{lat})\sin(\phi) + \cos(d_{long} - \theta)\cos(d_{lat})\cos(\phi)) - 100$$

subject to:

$$\begin{aligned}
FCc(g,\phi,\theta) &\leq g_{max}(w) && (10)\\
FRc(\phi,\theta) &\leq g_{max}(w) && (11)\\
h_{max} &\geq (FCt(h,\phi,\theta)+FRt(\phi,\theta)) && (12)\\
WFL(g,\phi,\theta) &\geq FRd(\phi,\theta) && (13)\\
g_{max}(w)+h_{max} &\geq FCc(g_{max}(w),\phi,\theta)+FCt(h_{max},\phi,\theta) &&\\
&\quad +FRc(\phi,\theta)+FRt(\phi,\theta) && (14)\\
g+h &\geq FCc(g,\phi,\theta)+FCt(h,\phi,\theta)+FRc(\phi,\theta)+FRt(\phi,\theta) && (15)\\
g &\geq FRca(\phi,\theta) && (16)\\
h &\geq FRta(\phi,\theta) && (17)\\
g &\leq g_{max}(w) && (18)\\
h &\leq h_{max} && (19)
\end{aligned}$$

where:

$$\begin{aligned}
WFL(g,\phi,\theta) &= g-FCc(g,\phi,\theta)\\
FRd(\phi,\theta) &= -w-\frac{a_0+a_1(EW_c+w)}{a_1}\\
&\quad +\frac{\sqrt{(a_0+a_1(EW_c+w+g))^2-2a_1D_{ra}(\phi,\theta)}}{a_1}\\
&\quad +desc_c+PAL_c+res_c
\end{aligned}$$

$$D_{ra}(\phi,\theta)=(R+alt)\arccos(\sin(a_{lat})\sin(\phi)+\cos(a_{long}-\theta)\cos(a_{lat})\cos(\phi))-100$$

3.6 Model 4

Previously, it was assumed that refueling occurred instantaneously at a point in space. In actual operations, the AR requires an air refueling track of approximately two to four hundred miles in length and takes approximately 1/2 hour to complete

[5]. Figure 9 shows a typical AR track [12]. The purpose of this model is to include an AR track in the formulation, and to account for the associated fuel costs of the AR activity.

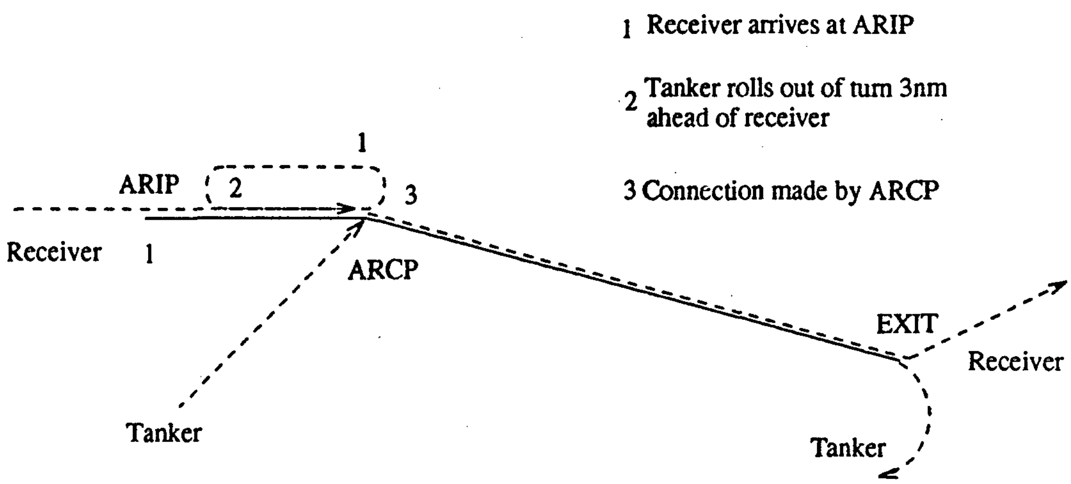

Figure 9. Typical Air Refueling Track

In order to use an AR track, the tanker and receiver either rendezvous somewhere prior to the AR track and fly there as a formation, or, this not being the case, the tanker flies to the Air Refueling Control Point (ARCP) [12:1-8]. If a delay is anticipated, the tanker will enter an orbit pattern at the ARCP but if not, the tanker will cross the ARCP and turn toward the receiver along a parallel heading that is offset a computed distance from the AR course [12:1-8]. Meanwhile, the receiver arrives at the Air Refueling Initial Point (ARIP) and initiates the rendezvous [12:1-8]. The receiver pilot then descends along the rendezvous heading toward the ARCP and plans to arrive there at the pre-determined Air Refueling Control Time (ARCT) [12:1-8]. At this point, the tanker will be on a reciprocal heading and, at a calculated range, begins a 180 degree turn in order to roll out on the rendezvous heading approximately 3 miles ahead of the receiver aircraft [12:1-8]. The receiver aircraft then closes to approximately 50 feet behind and slightly below the tanker and halts relative movement. This is know as precontact position [12:1-8]. Next, the receiver pilot carefully moves into position and connection is made with the tanker aircraft. This process should be completed by the ARCP [12:1-8]. The aircraft then fly toward the exit point. The physical connection between the aircraft is maintained until the required fuel transfer is completed or until a *bingo point* is reached without sufficient fuel transfer. The bingo point is a point between the ARCP and the exit point (150 miles for planning purposes) by which the receiver aircraft must have obtained the required onload or else it must divert to the alternate [12:1-8]. Hopefully, this option will not be exercised, but the option must be maintained because an aircraft cannot be put into a situation where potential fuel starvation would threaten the safety of the aircrew. Assuming that the fuel transfer is successful, both aircraft are cleared to the exit point and then to the next point along their route of flight.

There are numerous AR Tracks published for use above the Continental United States (CONUS) and off both coasts. There are also AR tracks published for use over Europe and their coast. Although most AR missions utilize published tracks, it is possible to create one for a specific mission with prior approval of Air Traffic Control (ATC) [17].

For modeling purposes, the AR track is simplified somewhat: It is assumed that the ARCP is the rendezvous point (θ, ϕ) and that the AR track extends for exactly 200 nautical miles along the great circle route from the ARCP to the destination of the receiver aircraft. At the exit point, the receiver continues on to its destination and the tanker returns to base. Also, the ARCP and the bingo point are considered the same. Figure 10 shows the model abstraction of the AR track.

The key elements necessary to include the AR track in the model are the latitude and longitude coordinates of the exit point, (e_{lat}, e_{long}). These coordinates can be determined through the use of spherical trigonometry. To do this, two spherical triangles are defined as shown in Figure 11. The large spherical triangle, ABC, includes the small one, $A_s B_s C_s$ on the right side. The lengths of the sides are measured by the angles they subtend with the center of the earth. This makes a spherical

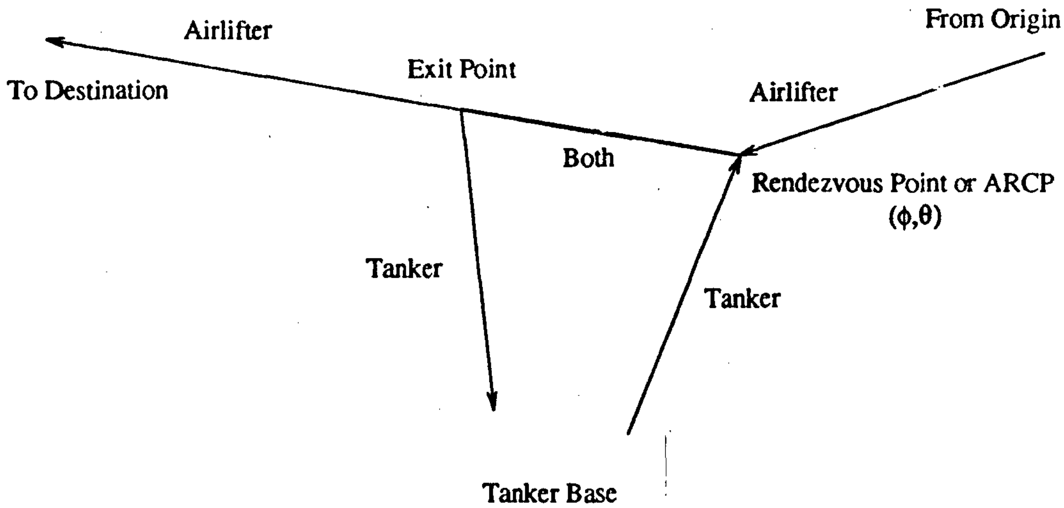

Figure 10. Model AR track

triangle easy to work with in latitude and longitude coordinates. By choosing the vertices of the triangles as shown in Figure 11, most of the sides, and one of the angles can be determined by inspection. For the big triangle, side $a = 90 - \phi$, side $c = 90 - d_{lat}$, and, since there are sixty nautical miles in one degree along the equator, a line of longitude, or any other great-circle arc, side $b = \frac{1}{60} D_{rd}(\phi, \theta)$. Also, angle $B = d_{long} - \theta$. For the small triangle, side a is shared with the big triangle and side $b_s = \left(\frac{1}{60}\right)(200)$. The quantity of interest is the (lat, long) coordinates of the exit point, shown in Figure 11 as the intersection of c_s and b. The latitude of the exit point is:

$$e_{lat} = 90 - c_s.$$

The longitude of the exit point for an East-to-West flight is:

$$e_{long} = \theta + B_s.$$

For a West-to-East flight, the exit point longitude becomes:

$$e_{long} = \theta - B_s.$$

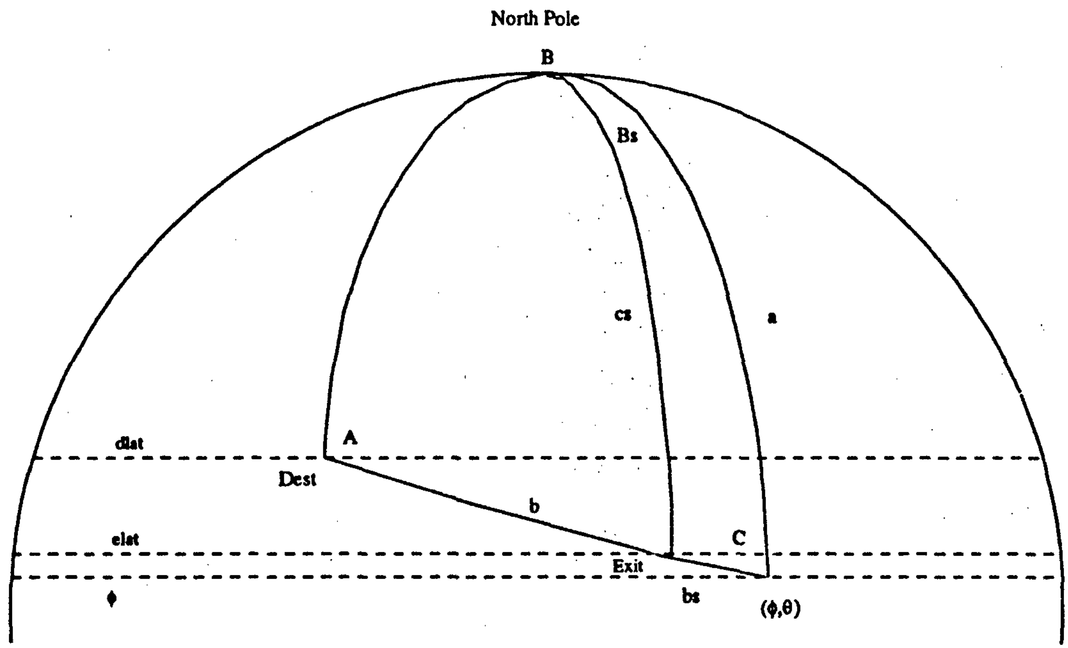

Figure 11. Triangles used to solve for the exit point

Unfortunately, c_s and B_s cannot be found by inspection; it is necessary to make use of the Law of cosines for spherical triangles,

$$\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(A)$$

to find these items [19:340]. First, the latitude coordinate, e_{lat}, is defined. The side

$$c_s = \arccos(\cos(a)\cos(b_s) + \sin(a)\sin(b_s)\cos(C))$$

requires angle

$$C = \arccos\left(\cos(c) - \frac{\cos(a)\cos(b)}{\sin(a)\sin(b)}\right)$$

from the big triangle. The use of the above two equations puts side c_s in terms of known quantities and, $e_{lat} = 90 - c_s$ gives the desired latitude coordinate. In order to find the longitude coordinate, e_{long}, it is necessary to find

$$B_s = arccos\left(\cos(b_s) - \frac{\cos(c_s)\cos(a)}{\sin(b_s)\sin(a)}\right)$$

which is already in terms of known quantities. Therefore, $e_{long} = \theta + B_s$ gives the longitude coordinate of the exit point. This step-through process and these equations are actually written into the model code in order to calculate an exit point for each rendezvous point the model generates.

Now that (e_{lat}, e_{long}) is available, the rest of the model can be tied together. In previous models, the distance from the tanker base to the rendezvous point was assumed to be the same as the reverse. This was a convenient result of the assumption that the AR took place at a point in space. Now that an AR track is defined, the new distance

$$D_{rb} = (R + alt)\arccos(\sin(t_{lat})\sin(e_{lat}) + \cos(t_{long} - e_{long})\cos(t_{lat})\cos(e_{lat})),$$

represents the distance from the exit point to the tanker base. This is illustrated in Figure 11. However, the two distances the model computes for the tanker flight, D_{br} and D_{rb} fail to account for the 200 nm AR track. This problem is corrected by simply adding 200 nm to D_{br}. Mathematically:

$$\begin{aligned} D_{br} &= (R + alt)\arccos(\sin(t_{lat})\sin(\phi) \\ &\quad + \cos(\theta - t_{long})\cos(t_{lat})\cos(\phi)) + 200. \end{aligned}$$

At this point, the model accounts for climb, cruise, descent, PAL, and reserve fuels for each aircraft over the entire flight including the AR track. However, the fuel burned along the AR track is calculated as though it were a normal cruise operation and this is not the case in reality. The AR maneuver itself increases the fuel consumption of both aircraft. The fuel consumption of the KC-135E is increased by 25 pounds per minute over cruise due to the increased drag of having the refueling boom extended [10:1A7-3]. The C-141B burns more fuel during the AR than in cruise flight for three reasons. First, flying behind and below the tanker subjects the receiver to the tanker's downwash. In order to maintain position, the receiver must be constantly "climbing" through this descending air at a rate of a few hundred feet per minute [9:9-54]. Second, the AR is done at a higher airspeed than normal cruise; 0.75 mach or 275 knots calibrated air speed, whichever is higher [9:9-54]. And lastly, the throttle movements and control inputs required to maintain position also tend to increase fuel consumption [5]. The actual fuel consumption for the C-141B while air refueling behind a KC-135 depends on the temperature, altitude and gross weight of each aircraft [9:9-54]. However, a number value of 300 pounds of fuel per minute was computed by using the charts in the performance manual [9:9-61].

For modeling purposes, the AR is assumed to last 30 minutes. This means the C-141 uses 9000 pounds of fuel during the modeled air refueling and this is included as a fixed cost. That 9000 pounds is the total fuel cost to the C-141 over the 200 nautical mile AR track. Therefore, D_{rd} must be redefined as beginning at the exit point instead of the rendezvous point, otherwise $FRc(\phi, \theta)$ would overestimate the required fuel by an amount equal to the cruise fuel for the AR track. From Model 3,

$$\begin{aligned} D_{rd}(\phi, \theta) &= (R + alt) \arccos(\sin(d_{lat}) \sin(\phi) \\ &\quad + \cos(d_{long} - \theta) \cos(d_{lat}) \cos(\phi)) - 100 \end{aligned}$$

now becomes

$$\begin{aligned} D_{rd} &= (R + alt) \arccos(\sin(d_{lat}) \sin(e_{lat}) \\ &\quad + \cos(d_{long} - e_{long}) \cos(d_{lat}) \cos(e_{lat})) - 100. \end{aligned}$$

The AR fuel cost must be represented in the formulation somewhere, so, in order to make the constraints work properly and avoid cluttering the objective function, the 9000 pound AR cost is added to $FRc(\phi, \theta)$.

For the tanker, no further manipulation of the flight distances is necessary because the refueling cost is given as 25 pounds per minute above cruise fuel flow. The function $FCt(\phi, \theta, h)$ represents the fuel consumed by the tanker from takeoff to the exit point, but an additional fuel cost of 750 pounds must be added to $FCt(\phi, \theta, h)$ for the AR track.

Overall, there were a number of details changed for Model 4 but the basic formulation is quite similar to Model 3. This concludes the changes for Model 4. It can be summarized by restating the model starting with the objective function:

$$\min V(g, h, \phi, \theta) = FCc(g, \phi, \theta) + FRc(\phi, \theta) + FCt(h, \phi, \theta) + FRt(\phi, \theta)$$

Where:

$$\begin{aligned} FCc(g, \phi, \theta) &= g + \frac{a_0 + a_1(EW_c + w)}{a_1} \\ &+ \frac{\sqrt{(a_0 + a_1(EW_c + w + g))^2 - 2a_1 D_{or}(\phi, \theta)}}{a_1} \\ &+ c_0 + c_1(EW_c + w + g) \end{aligned}$$

$$\begin{aligned} FCt(h, \phi, \theta) &= h + \frac{a_0 + a_1 EW_t}{a_1} \\ &+ \frac{\sqrt{(a_0 + a_1(EW_t + h))^2 - 2a_1 D_{br}(\phi, \theta)}}{a_1} \\ &+ d_0 + d_1(EW_t + h) + 0.75 \end{aligned}$$

$$FRc(\phi, \theta) = -w - \frac{a_0 + a_1(EW_c + w)}{a_1}$$

$$+\frac{\sqrt{(a_0 + a_1(EW_c + w + g))^2 - 2a_1 D_{rd}(\phi,\theta)}}{a_1}$$
$$+desc_c + PAL_c + res_c + 9.0$$

$$\begin{aligned} FRt(\phi,\theta) &= -\frac{a_0 + a_1 EW_t}{a_1} \\ &+\frac{\sqrt{(a_0 + a_1(EW_t + h))^2 - 2a_1 D_{br}(\phi,\theta)}}{a_1} \\ &+desc_t + PAL_t + res_t \end{aligned}$$

$$WFL(g,\phi,\theta) = g - FCc(g,\phi,\theta)$$

$$\begin{aligned} FRd(\phi,\theta) &= -w - \frac{a_0 + a_1(EW_c + w)}{a_1} \\ &+\frac{\sqrt{(a_0 + a_1(EW_c + w + g))^2 - 2a_1 D_{ra}(\phi,\theta)}}{a_1} \\ &+desc_c + PAL_c + res_c \end{aligned}$$

$$\begin{aligned} D_{or}(\phi,\theta) &= (R + alt)\arccos(\sin(o_{lat})\sin(\phi) \\ &+ \cos(\theta - o_{long})\cos(o_{lat})\cos(\phi)) \end{aligned}$$

$$\begin{aligned} D_{ra}(\phi,\theta) &= (R + alt)\arccos(\sin(a_{lat})\sin(\phi) \\ &+ \cos(a_{long} - \theta)\cos(a_{lat})\cos(\phi)) - 100 \end{aligned}$$

$$\begin{aligned} D_{br}(\phi,\theta) &= (R + alt)\arccos(\sin(t_{lat})\sin(\phi) \\ &+ \cos(\theta - t_{long})\cos(t_{lat})\cos(\phi)) + 200 \end{aligned}$$

$$\begin{aligned} D_{rb} &= (R + alt)\arccos(\sin(t_{lat})\sin(e_{lat}) \\ &+ \cos(t_{long} - e_{long})\cos(t_{lat})\cos(e_{lat})) - 100 \end{aligned}$$

$$
\begin{aligned}
D_{rd} &= (R + alt) \arccos(\sin(d_{lat}) \sin(e_{lat}) \\
&\quad + \cos(d_{long} - e_{long}) \cos(d_{lat}) \cos(e_{lat})) - 100
\end{aligned}
$$

Subject to:

$$FCc(g, \phi, \theta) \leq g_{max}(w) \tag{20}$$

$$FRc(\phi, \theta) \leq g_{max}(w) \tag{21}$$

$$h_{max} \geq (FCt(h, \phi, \theta) + FRt(\phi, \theta)) \tag{22}$$

$$WFL(g, \phi, \theta) \geq FRd(\phi, \theta) \tag{23}$$

$$
\begin{aligned}
g_{max}(w) + h_{max} \geq\ & FCc(g_{max}(w), \phi, \theta) + FCt(h_{max}, \phi, \theta) \\
& + FRc(\phi, \theta) + FRt(\phi, \theta)
\end{aligned} \tag{24}
$$

$$g + h \geq FCc(g, \phi, \theta) + FCt(h, \phi, \theta) + FRc(\phi, \theta) + FRt(\phi, \theta) \tag{25}$$

$$g \geq FRca(\phi, \theta) \tag{26}$$

$$h \geq FRta(\phi, \theta) \tag{27}$$

$$g \leq g_{max}(w) \tag{28}$$

$$h \leq h_{max} \tag{29}$$

3.7 Model 5

Up to this point, it was assumed that the effects of wind could be ignored. However, wind speeds at altitude may easily be on the same order of magnitude as that of the aircraft. This means that the wind may have a significant impact on fuel consumption and it is therefore desirable to include it in the model. The purpose of the fifth and final model is to account for the effects of wind.

Wind is the motion of an air mass relative to the ground. Once an aircraft leaves the ground and joins the air mass, it is caught up in a moving medium like a boat on a river. The velocity of the aircraft relative to the ground becomes the vector sum of the aircraft velocity relative to the air mass and the velocity of the air mass relative to the ground. This is illustrated in Figure 12.

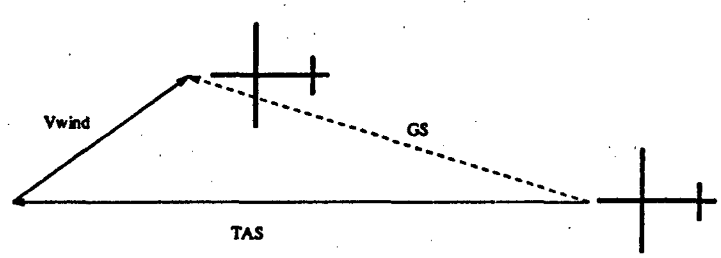

Figure 12. The wind speed and aircraft speed vectors

The magnitude of the aircraft velocity relative to the air mass is called the ***true airspeed*** (TAS) measured in knots, or nautical miles per hour. The term true airspeed is used to differentiate from other measures such as ***indicated airspeed.*** Indicated airspeed (IAS) is read on the airspeed instrument and, because air density decreases with altitude, TAS $\geq$ IAS. For modeling purposes, all aircraft airspeeds are considered true airspeeds. The direction of the aircraft velocity relative to the air mass is called the ***true heading.*** True heading is also the direction the nose of the aircraft is pointing, and it is measured in terms of the angle formed by the aircraft center line and a line of longitude. This angle is measured in degrees clockwise from north. Figure 13 illustrates this convention.

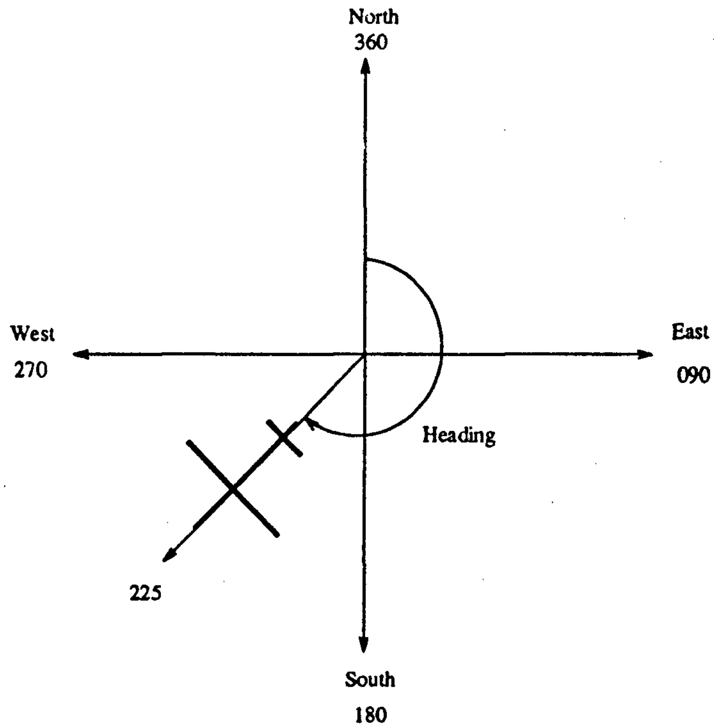

Figure 13. Heading Convention

Another way to measure heading is with a magnetic compass. The result is a *magnetic heading* and it is not, in general, the same as the true heading. The use of magnetic headings would unnecessarily complicate the model so only true heading is used.

The magnitude of the wind velocity, or wind speed, is also measured in knots. Its direction is given in degrees similar to true heading, but it is reported by weather agencies as the direction a weather vane would point. That is 130 degrees opposite the *heading* of the air mass. In the model, the actual heading of the air mass is used.

The velocity of the aircraft relative to the ground is called the *ground speed* and the path that the aircraft flies over the ground is called the *course.* When a pilot follows a particular course, such as a great circle arc between two points, the aircraft heading must be adjusted so that the ground speed vector is aligned with the course. Notice that when there is no wind, TAS = GS and the true heading is the same as the course.

For modeling purposes, it is assumed that the pilot adjusts the aircraft heading as necessary to fly on course, and that constant wind exists over the entire region of interest. The assumption of a constant wind over a several million square mile area is not as bad as it may seem. If it were possible to predict the velocity field of the wind exactly, then a true average wind could be determined. That true average wind would have the same effect on fuel consumption as the actual winds. Therefore, the constant wind used in the model should be the approximate mean for the area in question. Bordelon and Marcotte give a value of 263 degrees at 55 knots as an average wind for east–west flights at mid latitudes [3:26]. This value is converted to 083 degrees at 55 knots for use in the model.

In order to quantify the effect of wind on fuel consumption, it is important to remember that the model calculates fuel consumption based on the fuel mileage function, $NAM(GW)$. This fuel mileage function is totally unaffected by wind; that is, the aircraft gets the same number of air nautical miles per 1000 pounds of fuel at a given gross weight with a 100 knot headwind as it does with a 100 knot tailwind. The use of air nautical miles in place of ground nautical miles is acceptable for the no–wind case assumed in the previous four models. Therefore, the number of ground nautical miles per 1000 pounds of fuel as a function of gross weight and wind must be determined in order to upgrade the model.

In order to define this function, recall that the fuel consumed per unit time is a function of gross weight alone and not affected by wind. Therefore, the effect of wind on fuel consumption lies in the conversion of air nautical miles to ground nautical miles. If the aircraft is flying into a headwind, then the ratio of ground nautical miles to air nautical miles is less than one. The opposite is true in the case of a tailwind. This means that every point on the $NAM(GW)$ line is either raised or lowered by a factor of the ground speed to true airspeed ratio. To account for this, the intercept coefficients, a_0, and b_0 in the $NAM(GW)$ functions are multiplied by the ratio of the ground speed to the true airspeed. The number of ground nautical miles per 1000 pounds of fuel as a function of gross weight, GW, and wind velocity, V_{wind}, can be expressed as

$$NGM_{C-141B}(GW, V_{wind}) = \frac{GS}{TAS}a_0 + a_1 GW$$

for the C-141B and

$$NGM_{KC-135E}(GW, V_{wind}) = \frac{GS}{TAS}b_0 + b_1 GW$$

for the KC-135E tanker. This function is applied to the model by first computing a fuel mileage correction factor, F_i, for each of the five legs defined in Table 6. That is:

$$F_i = \frac{GS_i}{TAS_{C-141B}}, i = 1, 1a, 4$$

and

$$F_i = \frac{GS_i}{TAS_{KC-135E}}, i = 2, 3.$$

Next, the product of the fuel mileage correction factor and the intercept coefficient of

Table 6. Five legs of the problem

Leg	From	To
1	origin	rendezvous point
1a	rendezvous point	AR alternate
2	tanker base	rendezvous point
3	exit point	tanker base
4	rendezvous point	destination (via exit point)

the fuel mileage function is substituted for the intercept coefficient in the appropriate fuel consumed and fuel required functions. In leg 1, for example, where the C-141B flies from the origin to the rendezvous point, $a0$ is replaced by $F_1 a0$ in $FCc(\phi, \theta, g)$.

That is:

$$FCc(g,\phi,\theta) = g + \frac{F_1 a_0 + a_1(EW_c + w)}{a_1}$$
$$+\frac{\sqrt{(F_1 a_0 + a_1(EW_c + w + g))^2 - 2a_1 D_{or}(\phi,\theta)}}{a_1}$$
$$+c_0 + c_1(EW_c + w + g)$$

The true airspeed of each aircraft is given by the assumed flight conditions of mach 0.75 for the KC-135E and mach 0.74 for the C-141B at 31,000 feet. In converting mach number to true airspeed, standard temperature is assumed at altitude. This gives: $TAS_{C-141B} = 435$ knots and $TAS_{KC-135E} = 440$ knots [10]. The true airspeed is assumed to be constant for each aircraft. At this point, the only unknown quantities left are the ground speeds along each leg.

The geometry of the problem is helpful at this point. Figure 14 shows the TAS as the hypotenuse, or side a, of an oblique triangle while GS and V_{wind} form the other two sides b and c respectively. The length of the ground speed vector or side

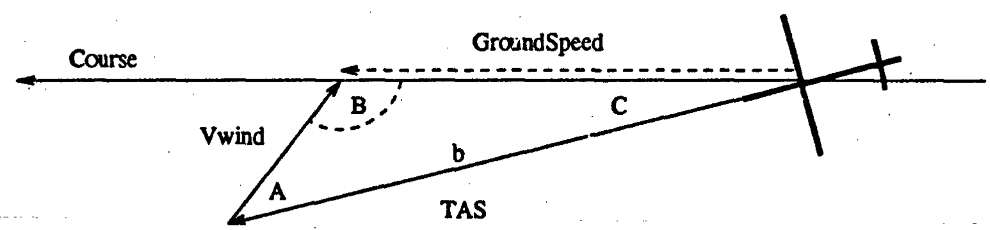

Figure 14. Ground Speed, Course, TAS, and Wind

a, can be found by applying the law of cosines [19:339]:

$$a^2 = b^2 + c^2 - 2bc\cos(A).$$

The only unknown in the above equation is angle A, but it takes some manipulation to find it. If H_c is the direction of the course in degrees and H_{wind} is the heading of the air mass in degrees then angle B is given by:

$$B = 180 - |180 - |H_c - H_w||.$$

Angle C is found by applying of the law of sines [19:339], $\frac{\sin(B)}{b} = \frac{\sin(C)}{c}$, in the following manner:

$$C = \arcsin\left(\sin(B)\frac{c}{b}\right).$$

Since the sum of the angles in a plane triangle is always 180 degrees,

$$A = 180 - B - C.$$

That gives angle A and thus the ground speed, $GS = a$, is determined by the above set of equations once H_c, the course, is known.

The great circle arc between two points cannot be flown with a constant heading. However, it can be modeled by defining the course as the local no–wind heading necessary to fly along the great circle arc. Figure 15 shows the initial heading at the beginning of a great circle arc, the final heading at the end of the arc, and the average heading for the entire arc. In this model, the course used in the ground

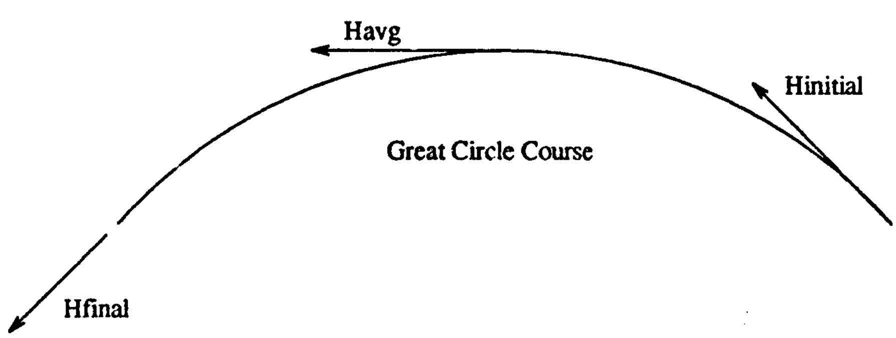

Figure 15. Arc showing H initial, H final and H avg

speed computations is defined as the average no–wind heading necessary to fly the great circle arc between the points in question. This means that the model uses an average heading and an average wind to find an average ground speed, which is used to correct the fuel mileage function for wind. At this point, the last mathematical step is to determine the average no-wind heading that defines the course.

The initial heading used to follow a great circle course between two points is given by the heading equation [15]:

$$H(L_1,\lambda_1,L_2,\lambda_2) = \arccos\left(\frac{\sin(L_2) - sin(L_1)cos(\frac{D(L_1,\lambda_1,L_2,\lambda_2)}{60})}{sin(\frac{D(L_1,\lambda_1,L_2,\lambda_2)}{60})cos(L_1)}\right).$$

Where, L_1 is the latitude coordinate of point 1, λ_1 is the longitude of point 1, and $D(L_1,\lambda_1,L_2,\lambda_2)$ is the great circle distance between the points 1 and 2. The final heading into point 2 can be determined by finding the initial heading necessary to fly from point 2 to point 1, $H(L_2,\lambda_2,L_1,\lambda_1)$, and then either adding or subtracting 180 degrees in order to find the reciprocal heading. (See Figure 15.) Once this is done, and both the initial and final headings are in hand, the heading that defines the course, H_c, is the simple average of the two:

$$H_c = \frac{H_{initial} + H_{final}}{2}.$$

The course, H_c, allows computation of the average ground speed, GS, and thus the fuel mileage correction factor, F_i. This factor then updates the fuel consumed and fuel required functions as previously described. Aside from the internal changes in the fuel required and fuel consumed functions, the only other change to the model is the mathematical overhead necessary to define F_i. The rest of the model appears identical to Model 4, and is not repeated here.

3.8 *Maximizing Allowable Cabin Load*

Model 5 can be easily modified to handle the related problem of maximizing Allowable Cabin Load (ACL). Maximizing ACL is the other way to state the flight planner's problem, and is of frequent concern to the flight planners at Air Mobility Command Headquarters [4]. The allowable cabin load may be considered the maximum cargo weight the aircraft can carry on a given mission. For modeling purposes, the cargo weight can be stated as

$$w = MaxTO_c - EW_c - g, \tag{30}$$

that is, the difference between the maximum take-off weight, also assumed to be the maximum gross weight, and the sum of the empty weight and the fuel. According to equation (30), the upper bound on ACL would occur when $g = 0$. In reality, ACL may be limited by structural considerations instead of simply by fuel [5]. But, it is unlikely that g can be made small enough to make w unreasonable with the mission profile under consideration.

To make these modifications, the first thing to change is the definition of cargo weight, w. In Model 5, w is simply a constant parameter. Since it now becomes the quantity of interest, it is replaced by equation (30). The only other necessary change is to the objective function. The objective is now to maximize ACL, but the SQP code only does minimization. The easy answer to this problem is to minimize $-w$. Therefore, the new objective function becomes:

$$Minimize \quad V = EW_c + g - MaxTO_c.$$

The constraint set is unchanged.

With these changes, the model reports $-w$ and the optimal values of the decision variables, g, h, ϕ, and θ required to move w pounds of cargo. The results can be verified by setting w in Model 5 to whatever "maxACL" found w to be, and then running Model 5. If the values of the decision variables are identical, then maxACL is giving the correct result.

3.9 Modeling Summary

Now that the last model has been covered, it is appropriate to summarize the modeling effort to this point. The first section of this chapter covered Yamani's formulation of the flight planner's problem as an NLP. The next section covered the changes for Model 2 which included changing the aircraft data and the mission profile. These are simply changes in constant values within the model and do not affect its structure. Model 3, on the other hand, made significant changes in that several constraints were redefined in terms of fuel instead of distance and a new constraint was added to keep the rendezvous point within range or the AR alternate. Also, fuel costs for climb, descent, landing, and reserve were incorporated into the appropriate fuel functions. In the fourth and fifth models, an AR track and the effects of a constant wind were accounted for. Model 5, containing all of the enhancements, was then modified to solve the related problem of maximizing ACL. After each model was formulated, it was solved using SQP and the results are compiled in Chapter 4.The benefits of taking this incremental approach to the modeling process were that the de-bugging process was much easier and the numerical results of each model can be compared to show the relative effectiveness of each modification.

IV. Results

4.1 Overview

The purpose of this chapter is to discuss the results obtained through this research effort. General results, common to all of the models, begin the presentation. Then, specific results for each model are given. This includes a comparison of Model 1 results with those of Yamani and Coffman. The results of Model 5 and maxACL are also compared. Finally, a comparison of an actual flight plan with the numerical results from Model 5 and maxACL is made.

4.2 General Results

The following results were obtained by running the models on a Sun workstation. Computational times are on the order of one second for all cases. In practice, SSQP is a reliable and highly accurate means of solution. During the programming and debugging process, no problems were encountered with SSQP itself. It was apparently insensitive to starting point conditions because the same output was obtained when different starting points were used. It is important to note that SSQP does not require a feasible starting point. SSQP is also very accurate. The default setting for the user-selectable final accuracy is 10^{-7}. This means, for any objective function $f(x)$,

$$f(x) \leq (1+\epsilon)f(x^*),$$

and for the solution vector x,

$$R(x) \leq \epsilon,$$

where x^* is the optimal solution and $\epsilon = 10^{-7}$, the objective function value will be within 10^{-7}% of the actual minimum and the solution vector will be within a radius of 10^{-7} about the actual solution [21]. In the context of these models, SSQP is 10,000 times more accurate than needed to find the total fuel cost to the nearest pound and it finds the rendezvous point to the nearest one-half inch. Therefore, model inputs, parameters, and formulation are the determining factors in output accuracy.

The output generated by SSQP contains more than just the values of the decision variables and the objective function. The value of each constraint and the approximations of the Lagrange multipliers are both useful sets of data for analyzing the solutions. The constraint values reported by SSQP are either less than zero indicating a violated constraint, equal to zero indicating a binding constraint, or greater than zero which indicates a non-binding constraint. For each binding constraint, there is a non-zero Lagrange multiplier. The value of this multiplier in the program output is the approximate number of units by which the objective function would decrease if one more unit of the constrained resource were available. The program output is given for each model along with the model code in Appendix B through F.

Each problem formulated in Chapter 3 was solved successfully. All solutions are feasible and the results of each model are operationally reasonable. The numbers are presented in the tables to follow. Table 7 gives the latitude and longitude coordinates of the various places used in the model. The coordinates of the locations used in Models 2 through 5 were obtained from the computer flight plan and from the Flight Information Handbook as discussed in Chapter 3. The other coordinates were reported by Yamani along with his results [27:789].

4.3 Model 1 Results

Model 1 is an exercise in solving Yamani's formulation with SSQP. This was done once for each of Yamani's five cases. Table 8 lists the results given by Yamani and Coffman as well as those obtained with Model 1. The data attributable to Yamani and Coffman is denoted by the "Y" in the first column. The "Onload" and "Total" numbers come from Coffman's thesis [7:36]. The other numbers reported in the "Y" rows are from Yamani's article in *Operations Research* [27:799].

Table 7. Approximate Airfield Locations

Location		Latitude	Longitude
Aviano Italy	AvI	46.03 N	12.60 E
Azores Islands	AZ	37.00 N	25.00 W
Delaware	Del	38.00 N	75.00 W
Egypt	EG	30.00 N	28.00 E
England	GBr	52.00 N	0.00
Germany	Ger	50.00 N	10.00 E
Iceland	Ice	65.00 N	20.00 W
Lajes, Azores	Laj	38.70 N	27.10 W
New Jersey	NJ	40.00 N	75.00 W
North Carolina	NC	35.00 N	78.00 W
McGuire AFB NJ	McG	40.02 N	74.60 W
Puerto Rico	PR	18.00 N	66.00 W
Saudi Arabia	SAr	25.00 N	47.00 E
Turkey	Tky	40.00 N	30.00 E

In Tables 8 and 10, the code in the first column indicates the source for that row. The next three columns describe the mission profile. The rest of the columns are self-explanatory except that the "star" superscript indicates values at optimality. Notice that all of the Model 1 results match Yamani's quite closely, and there is a pattern of lower objective function values than Yamani and Coffman report.

Another noticeable pattern is the location of the rendezvous point at optimality. All models tend to converge to the coordinates of the tanker base if it is enroute to the destination. If not, then they usually converge to a point on the boundary of the feasible region near the tanker base.

4.4 Model 2 Results

Model 2 is identical to Model 1 except that some of the constant data is changed to reflect the different aircraft and mission profile. The optimal rendezvous point occurs over the tanker base and Model 2 is the baseline for comparison with the other models that follow.

Table 8. Results: Model 1 and Yamani

Origin, Destination, & Tanker Base				Cargo Wt.	Optimal AR Point (ϕ^*, θ^*)		g^*	h^*	Onload	Total
Y)	NJ	Tky	PR	200.00	40.00 N	49.00 W	113.52	285.76	136.70	399.28
1)	NJ	Tky	PR	200.00	39.39 N	48.88 W	140.60	251.00	115.66	392.60
Y)	Ger	NC	Ice	200.00	65.00 N	20.00 W	79.43	131.60	130.54	211.03
1)	Ger	NC	Ice	200.00	65.00 N	20.00 W	64.29	142.08	142.08	206.37
Y)	Del	SAr	AZ	100.00	37.00 N	25.00 W	137.14	150.33	144.46	287.47
1)	Del	SAr	AZ	100.00	37.00 N	25.00 W	105.17	171.89	171.86	277.03
Y)	Del	EG	PR	200.00	36.00 N	42.00 W	118.71	309.40	152.51	428.01
1)	Del	EG	PR	200.00	35.27 N	42.72 W	115.14	302.61	156.50	417.75
Y)	NC	GBr	Ice	200.00	63.00 N	29.00 W	129.88	73.69	54.54	203.58
1)	NC	GBr	Ice	200.00	62.59 N	26.77 W	130.00	70.30	52.23	200.30

Table 9. Legend for Tables 8, and 10

Y)	Results reported by Yamani and Coffman
1)	Results Obtained Using Model 1
2)	Results Obtained Using Model 2
3)	Results Obtained Using Model 3
4)	Results Obtained Using Model 4
5)	Results Obtained Using Model 5
6)	Results Obtained Using maxACL
n/a	Data not available

NOTE: weight given in units of 1000 pounds

4.5 *Model 3 Results*

Model 3 takes into account the climb, descent, reserve, and approach fuel costs. It also requires the airlifter to be within range of the destination or the alternate at all times. This is a significant increase in detail over Model 2. Since all of these factors tend to increase the fuel consumption, it is good to see the objective function value of Model 3 is higher than Model 2. The optimal rendezvous point no longer occurs overhead of the tanker base. This is because the feasible region has been

Table 10. Results: Models 2 thru 5, Including maxACL

Origin, Destination, & Tanker Base				Cargo Wt.	Optimal AR Point (ϕ^*, θ^*)		g^*	h^*	Onload	Total
2)	AvI	McG	Laj	50.00	38.70 N	27.10 W	53.36	65.44	65.41	118.77
3)	AvI	McG	Laj	50.00	40.34 N	26.75 W	119.95	22.11	13.37	142.05
4)	AvI	McG	Laj	50.00	40.11 N	25.11 W	115.59	35.27	20.41	150.86
5)	AvI	McG	Laj	25.00	40.48 N	24.54 W	113.53	54.97	39.65	168.50
6)	AvI	McG	Laj	73.23	53.32 N	18.96 W	97.18	100.98	56.78	198.16
5)	AvI	McG	Laj	73.23	53.32 N	18.96 W	97.18	100.97	56.78	198.16

reduced in size by the additional fuel costs and by the divert constraint. The tanker base no longer lies within the feasible region.

4.6 Model 4 Results

The addition of the AR track and AR fuel costs increased the objective function value as expected. The rendezvous point, (ϕ^*, θ^*), has moved still further back toward Aviano because the tanker is now flying a triangular pattern and the AR takes it back toward the tanker base. The exit point is on the "other side" of the tanker base. The ground track is similar to what is shown for Model 5 in Figure 16. The divert base constraint is still binding. Notice that g^* has decreased and the onload has increased.

4.7 Model 5 Results

The objective function value for Model 5 is higher than that of Model 4 because of what amounts to a headwind. The airlifter fights the wind over its entire route while the tanker enjoys a tailwind for only its final leg. The optimal rendezvous point has moved further back toward Aviano and the divert constraint is binding. Also, if the wind velocity is set to zero and the cargo weight set to fifty thousand pounds, then an identical result to Model 4 is obtained.

Model 5 represents the highest level of detail achieved for the NLP model. Such detail is not without a price however. In order to build a more detailed model, more computer code is required. Model 3 has 33% more lines of code than Model 2. Model 4 tops five hundred lines of code and is another 12% larger than Model 3, while Model 5 betters them all with over nine hundred lines of code. A point of diminishing returns will eventually be reached in any modeling effort, and Table 11 is an attempt to gauge whether or not that happened here. Table 11 gives an idea of how much change was produced by Models 2, 4, and 5, using Model 2 as the baseline. The difference in the objective function is the percent increase over the previous model. The cumulative difference is the percent increase in the model's objective function relative to Model 2. The net movement of the rendezvous point is the movement from the previous model.

Table 11. Effects of changes for Models 3 thru 5

	Difference in objective function	Cumulative difference	Net movement of AR point, nautical miles
Model 3	19.6%	19.6%	100
Model 4	6.2%	27.0%	77
Model 5	11.7%	41.9%	34

4.8 Maximizing ACL

When maxACL, the modified version of Model 5 that finds the maximum allowable cabin load for a given mission, was run the objective function increased only 18% while the amount of cargo carried went up by a factor of three from Model 5. This indicates that maxACL is preferable when the amount of cargo to be carried is greater than one plane-load. For verification, Model 5 was run with the optimal cargo value from maxACL as the weight input. When this was done, Model 5 produced exactly the same result. Notice that the AR point is now very close to the great-circle course between Aviano and McGuire and that g^* is just enough to get the aircraft to the divert base. The allowable cargo load is limited by the location of the divert base. If the divert base is changed to England, the objective function value increases to 78,708 pounds of cargo.

4.9 *Comparison with the Computer Flight Plan*

A Computer Flight Plan (CFP) for an actual mission from Aviano to McGuire was obtained. Unfortunately, it is just for the C-141B and the matching flight plan for the tanker was unavailable. The computer flight plan calls for an AR near the coast of Portugal with an onload of 80,000 pounds from a KC-10. The route of flight is far from great circle between Aviano and the AR track, since it has the airlifter going through the Straits of Gibraltar. From the exit point to McGuire, the route becomes a better approximation of a great circle arc. This is illustrated in Figures 16 and 17 which compare the ground tracks of the CFP to that of Model 5 and to maxACL respectively. The following changes are made to Model 5 and

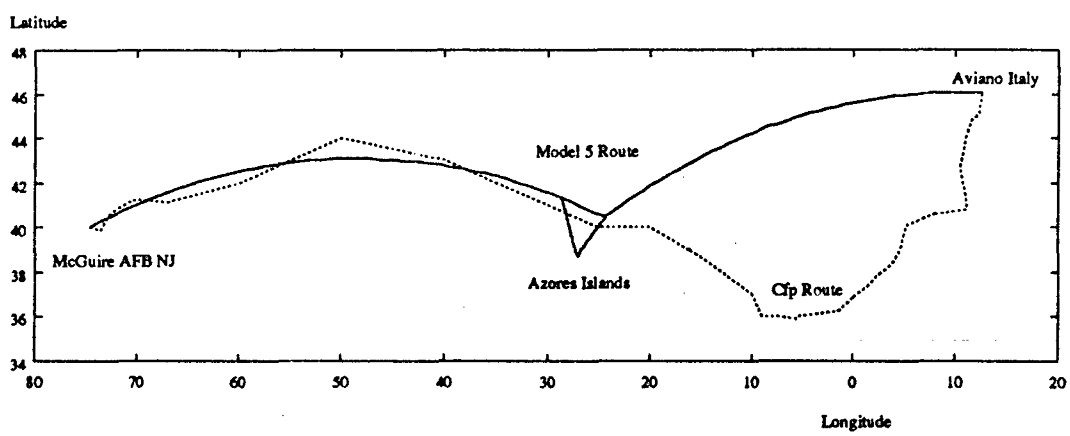

Figure 16. Ground Tracks for Computer Flight Plan and Model 5

maxACL in order to match the conditions of the CFP:

- The cargo weight, w, is changed in Model 5 from 25,000 pounds to 23,000 pounds. Of course, this did not apply to maxACL.

- The required reserve at the AR divert base is raised from 6.7 to 10.3 thousand pounds in both models.
- The reserve at the destination is increased from 6,700 to 26,437 pounds in both models to match the CFP. Remember, the models assume that no weather alternate is necessary at the destination. However, the CFP has one. To account for this, the landing fuel from the CFP is used as the reserve in the model. This works correctly because the landing fuel includes the fuel necessary to reach the weather alternate with a minimum reserve.
- The wind was changed to the approximate average over the CFP route. This value was found to be 274 degrees at 70 knots. It is changed to the "wind heading" used in the model by subtracting 180 degrees to give 094 degrees at 70 knots.

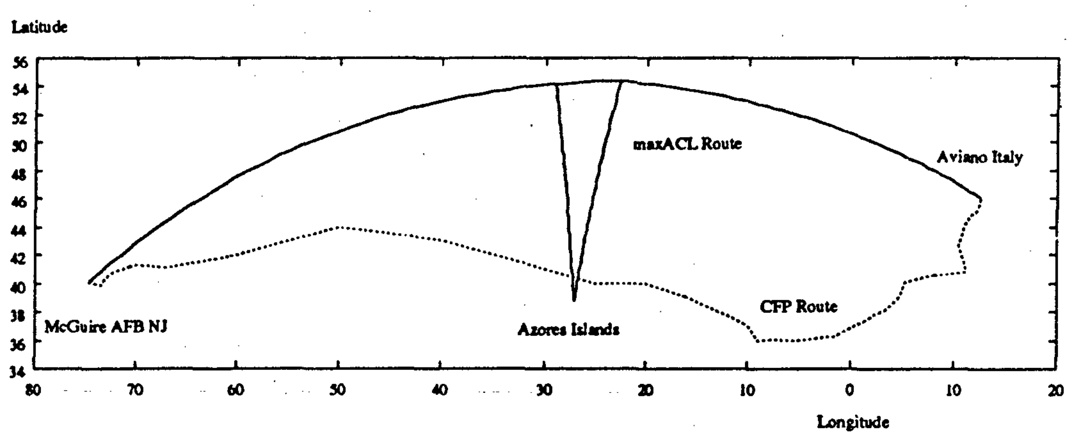

Figure 17. Ground Tracks for Computer Flight Plan and maxACL

Table 12 gives the numerical results obtained when Model 5 and maxACL are run with the adjustments listed above. Compared to the Computer Flight Plan, Model 5 results in the same amount of cargo transported for a 20% savings, or 31,163 pounds less fuel burned. Also, the route of flight determined by Model 5 is 20% shorter than the route given by the CFP. The results of maxACL were even more dramatic.

the amount of cargo more than doubled, but the fuel consumption for the transport dropped by almost as much as in Model 5.

Table 12. Comparison of Model 5 and maxACL with Computer Flight Plan

	CFP	Model 5	maxACL
ARCP Coordinates	37.55 N 1.22 W	40.46 N 24.32 W	54.37 N 22.80 W
Distance to ARCP	1554	1637	1435
Distance from exit to McG	2708	1877	1827
AR track length	228	200	200
Total distance	4490	3714	3462
Initial fuel	129.5	119.07	111.97
Onload	80.0	59.26	66.53
Total fuel burned by C-141	183.6	151.89	152.06
Cargo Carried	23.0	23.0	58.44

4.10 Summary

The SSQP results for Model 1 matched Yamani and Coffman quite well. That means the formulation is correct and the solution method is working properly. Model 2 is a small step from Model 1 because the only differences occur in the model parameters. Model 2 forms a baseline for comparing the effects of modifications to the later models. Models 3, 4, and 5 showed the increasing tendency of the objective function as more things were considered. MaxACL shows the model's flexibility by solving the related, and possibly more important, problem of maximizing ACL and doing it with only minor changes. Applying Model 5 to the results of maxACL and getting the same answer shows that maxACL is working correctly. The comparison with the CFP shows that the models produce realistic numbers. It also shows that significant improvements in efficiency are possible when the best AR point can be chosen and great circle flown to and from it.

V. Conclusions and Recommendations

5.1 Executive Summary

From the outset, this research was concerned with the extension of Yamani's work by removing the major simplifying assumptions and thereby creating a new analysis tool for solving the flight planner's problem. This was accomplished by the models described in Chapter 3.

The first step was to solve Yamani's formulation with sequential quadratic programming. This was done by the first model, Model 1, and the results are very similar to those reported by Yamani and Coffman. Then, Model 1 was enhanced and reformulated through a series of modeling iterations, culminating in Model 5 and maxACL. Although Yamani provided the mathematical underpinnings, Model 5 represents a vast increase in operational realism. The version of Model 5 referred to as maxACL, is an application of NLP modeling to a completely different, and perhaps much more important problem. Yamani did excellent work on the first part of the flight planner's problem, minimizing fuel cost for a fixed cargo weight, but Model 5 takes that work much further. MaxACL handles the second part of the flight planner's problem, maximizing the amount of cargo carried c a single mission; a problem not addressed by Yamani or Bordelon and Marcotte.

Model 5 finds the optimum rendezvous point and initial fuel for both the KC-135E tanker and the C-141B transport when given the airbase locations, cargo weight and average wind. MaxACL finds the maximum allowable cabin load for the C-141 as well as the rendezvous point and initial fuels given the airbase locations and average wind. In doing so, both versions consider:

- C-141B and KC-135E cruise performance at recommended mach numbers,
- fuel consumed in the climb, descent, approach, and landing phases of flight,
- AFR 60-16 requirements for fuel reserve,

- an air refueling track 200 nautical miles in length,
- the effect of an average wind.

The results of Model 5 and maxACL were compared with a computer flight plan for an actual mission. The comparison shows that a savings in fuel and an increase in cargo are possible when the results of Model 5 and maxACL are applied.

The fuel savings indicated by Model 5 is 31,710 pounds, or a little over $3400 at the current government price of $0.70 per gallon of jet fuel [1]. Even bigger savings are possible with the maxACL results because they show that the cargo weight can be doubled. By doubling the cargo weight, and thus saving an entire trip, fuel savings on the order of 300,000 pounds and $32,000 could be realized.

5.2 Overall Conclusions

Unfortunately, this research did not uncover a heuristic solution to the problem of minimizing total fuel cost with a fixed cargo weight. Rather, the following set of observations is offered:

- In the case of fixed cargo weight, the refueling will not take place on the great circle route between the origin and the destination unless the tanker base is exactly enroute. In other words, the receiver will "go out of its way" to the rendezvous point.
- If the tanker base is enroute, that is, within the feasible region, the model will usually converge so that the rendezvous point is either directly overhead or very close to the tanker base. This shows the inherent disadvantage of AR over landing for fuel. In Models 1 and 2 the result is essentially "land for fuel" when the rendezvous point is the tanker base. For the other models that take climb and descent into account, this calls for an "up and down" mission by the tanker.

- Increasing distance by 100 nm has about 30 times the effect on fuel consumption as increasing the cargo or fuel weight by 1000 pounds. From the standpoint of fuel consumption, the distance, and hence the location of the refueling point, is more important than the initial fuel.

The location of the optimal rendezvous point is interrelated with the problem geometry, aircraft performance, and other factors such as wind. The initial fuel is of less consequence and also interrelated with these factors. Therefore, when the goal is to minimize total fuel costs with a fixed cargo weight less than the maximum amount that one airlifter can carry, the use of math programming is necessary.

The application of Model 5 to the problem of maximizing allowable cabin load provides a more complete look at the flight planner's problem, and leads to the most important results of this research. If the overall goal is to save money, closure time, and airframe time on the airlifter, then maxACL is usually right model to use. Minimizing fuel as in Model 5 is only appropriate in the isolated case of a one-time airlift with a total cargo weight less than or equal to "one plane load." Although maxACL and Model 5 agree when the cargo weight approaches its maximum value, Model 5 can only find that maximum through trial and error. The maximum ACL is a critical piece of information when there is to be a flow of cargo because it allows the planners to minimize the number of sorties. Minimizing the number of sorties shortens the closure time, or time required to move all of the cargo, and will minimize the total cost. An example of this is apparent in Table 10. Notice that maxACL allows the C-141 to carry three times more cargo than assigned in Model 5 for only about 18% more fuel.

When planning a mission with a fixed cargo weight less than the maximum capacity of the airlifter, it would be best to run Model 5 once for each tanker base available, choose the lowest cost mission, and flight plan as closely as possible to the model result. In the case where a published AR track does not exist near the rendezvous location called for by the model, the additional fuel cost of using a published track should be weighed against other considerations. This is the most likely result in general, but Model 5 will provide a useful lower bound to the mission cost. The initial fuels should then be determined by standard means.

In the case where a flow of cargo is to be optimized, the model maxACL can find an upper bound on the amount of cargo each aircraft can carry. From there, the airspace usage can be determined as in the other case. This will result in a number of identical sorties to carry the bulk of the cargo. The last fraction of a plane-load can be optimized by applying Model 5. If maxACL is not available, there are heuristics that will help in planning such a mission. When ACL is to be maximized, the airlifter should never go out of its way for fuel. Therefore, a maxACL mission will normally follow the great-circle route from origin to destination. However, if strong winds exist, the great-circle route may not be the most efficient. The best point to refuel would be at the point where unrefueled flight to the destination just becomes feasible. This point will be subject to restrictions imposed by alternate bases and maximum onload by the tanker. This is essentially the way such AR missions are currently planned. However, maxACL would be helpful to the mission planner by providing a useful upper bound on the amount of cargo that can be carried and a quick estimate of the required onload and rendezvous point.

5.3 Recommendations for Further Research

This research has left many avenues unexplored and room for improvement in the models and methods used here. The strongest recommendation is that further work on the flight planner's problem be sponsored by, or at least coordinated with Air Mobility Command Headquarters. It is the opinion of the author that this research, and all following research in this area will be useful to the analysis branch at HQ AMC.

5.3.1 Further Enhancements. Model 5 did not encounter any fundamental limits of Yamani's formulation or the SSQP solution method. It certainly does not take into account all of the factors that flight planners must consider. Therefore quite a bit more research can be done by simply extending Yamani's formulation past Model 5.

One enhancement, that should prove to be fairly easy, is to consider the limitations imposed by the maximum crew duty day. The maximum length of time that an aircrew can remain on duty is limited by regulation and can become a problem for the mission planner [4]. In order to handle this, define two variables, T_c and T_t, to be the total duty time for the cargo aircraft and tanker aircrews respectively. There will be a minimum value for both T_c and T_t defined by the necessary ground preparation time prior to departure and after arrival. These can be considered either fixed or perhaps some function of cargo weight. The flight time for the mission can be determined by dividing the length of each leg by the ground speed on that leg and then adding the leg times. The effect of an AR alternate and a weather alternate at the destination of each aircraft should also be considered in calculating total flight times. This will generate two constraints of the form $T - T_{MAX} \geq 0$.

Another fairly simple improvement is the addition of a weather alternate at the destination of each aircraft. This can be done in a manner analogous to the AR divert developed in Model 3. As a minimum, two legs and two new constraints would be created.

Removal of the constant-altitude assumption would be more challenging, but it would make the model more realistic. It would be unusual for an actual AR flight to take place at a constant altitude for two reasons.

First, the altitude range for the AR maneuver is typically 24,000 to 25,000 feet [5]. At this lower altitude, the receiver aircraft has more power available thus making it easier for the pilot to maintain position during the AR. The AR maneuver is not a trivial exercise in flying. It takes a great deal of skill and practice on the part of the receiver pilot to successfully complete [5]. Unfortunately, the extra power available at lower altitudes comes at a price of increased fuel consumption. The fuel mileage for any aircraft increases with altitude and a substantial increase in efficiency is gained by flying at a high altitude such as 39,000 feet. To take advantage of this, an airlifter on an AR mission usually climbs to a fairly high altitude enroute to the Air Refueling Initial Point [5]. Once the airlifter arrives at the ARIP it descends to the AR altitude, refuels, and then climbs back to a high cruising altitude [12]. Depending on the distance to the AR track, the tanker either climbs to a high cruising altitude if the AR track is distant, or it only climbs to the AR altitude if the AR track is not far away. On the return leg, the tanker climbs to a higher altitude for cruise if the distance is large enough. In any case, the choice of altitudes is limited by the Instrument Flight Rules conventions for altitude assignments.

The second reason that long missions do not ordinarily take place at constant altitudes is Air Traffic Control (ATC) and the air route structure. In order to maintain separation of aircraft, ATC often directs changes in altitude. Also, additional restrictions on altitude can occur due to the North Atlantic (NAT) tracks and other airspace considerations [5].

For the purpose of flight planning, altitude assignments are based on rules or "if-then" conditions. It would seem logical to put "if-then" conditions into the program to handle the non-constant altitudes. Unfortunately, this could lead to trouble. Each different altitude to be modeled would require different set of NAM coefficients for each aircraft. If the coordinates of the rendezvous point are still to be the decision variables that define the routes taken by the aircraft, then adding if-then conditions concerning altitude may cause discontinuity problems with the objective function. If there were discontinuities in the objective function, then the derivatives would not exist at some points and one of SSQP's necessary assumptions would be violated. It is possible that SSQP would fail in this case.

The great-circle arc assumption could be relaxed somewhat. Route restrictions due to air traffic control, political considerations, NAT tracks, and other reasons can be taken into account. Consider the computer flight plan example. If the airlifter were required to fly through the Straits of Gibraltar, then one way to handle this is to define a new origin point as the point where the airlifter exits the straits. Minimum and maximum fuel conditions would be applied to this new origin. The minimum fuel aboard the airlifter at the straits would be equal to the minimum necessary to fly from there to the AR divert base. The maximum would be the amount the airlifter would have if it were to take off with full tanks. With the origin redefined and appropriate limits on the "initial fuel," the model could then be solved normally. This modification might prove to be more difficult in the case of maxACL.

The requirement of following certain "tracks" could be accounted for by cleverly defining constraints on ϕ and θ so that the ground track would approximate the desired track. When these modifications are attempted, it would be quite helpful to build some kind of graphics module so that the ground track determined by the program can be seen. The software package GNUPLOT, could be prove quite useful for this.

5.3.2 Applications. Once an NLP flight planning optimization code of sufficient detail, such as Model 5 and maxACL, or perhaps some further enhanced version is available, then the models could be used for the analysis of other related issues. For instance, these models could be used to find the best combination of aircraft type and airbase location in order to maximize cargo carried or minimize cost. Bordelon and Marcotte did much of their work in this problem area. Model 5 and maxACL could be run repeatedly to pick the best combination of tanker aircraft and tanker base for a given airlift scenario. The model could be treated like a simulation or, a simulation program could be built around the model in a manner similar to the work of Bordelon and Marcotte. This is only recommended for someone with real programming talent and a deep understanding of FORTRAN and SLAM.

Another interesting application of these models is the analysis of tanker basing and deployment. This is an issue that could become critical to an airlift effort when forward bases are limited or unavailable. The models could also be used to examine the recent airlift operations in support of Desert Shield, Desert Storm, and the effort in Somalia. Such an analysis would help identify the key factors that limit the effectiveness of such airlift operations.

In conclusion, this research has shed some light on the use of NLP to solve the flight planner's problem. However, the greatest benefits will come from the further research that is suggested.

Appendix A. Schittkowski's Sequential Quadratic Programming Algorithm

The purpose of this appendix is to outline K. Schittkowski's implementation of Sequential Quadratic Programming (SSQP). Sequential Quadratic Programming is a method of solving general non-linear programs of the form:

$$
\begin{array}{rll}
\text{Minimize} & f(x) & \\
\text{Subject to:} & g_j(x) = 0, & j = 1, \ldots, m_e \\
& g_j(x) \geq 0, & j = m_e + 1, \ldots, m \\
& x_l \leq x \leq x_u &
\end{array}
$$

where x is a vector of n real-valued decision variables. The values that x can take on are limited by the lower and upper bounds, x_l and x_u respectively. The problem has a total of m constraints, of which the first m_e are equality constraints. For typical problems, the constraints may have to be rewritten so that their right hand side is zero.

According to Schittkowski, SSQP will solve a general non-linear program under the following assumptions:

- The problem functions, including the objective function and the constraints, must be continuously differentiable over the range $x_l \leq x \leq x_u$ [22:486].
- The problem is small. Although SSQP has solved problems of up to 100 variables, it is limited by the storage capacity of the hardware and the ability of the quadratic programming subroutine to handle large problems [22:486].

Under these conditions, SSQP gives superior performance in terms of reliability and speed. In testing done by Schittkowski, SSQP was significantly faster than all other codes tested [22:498].

A.1 Definitions

Before the algorithm is presented, a few definitions are in order. The gradient of a function $f(x)$ is given by its commonly used notation

$$\nabla f(x) = \left(\frac{\partial}{\partial x_1} f(x), \ldots, \frac{\partial}{\partial x_n} f(x)\right)^T$$

for a vector of variables x [23].

Assuming the function $f(x)$ is twice-differentiable, the elements of the Hessian matrix **H**, can be written in the form

$$\nabla^2 f(x) = \left(\frac{\partial^2}{\partial x_i \partial x_j} f(x)\right)$$

meaning the second partial derivatives of $f(x)$ with respect to x_i occur along the diagonal while mixed partials occupy the rest of the matrix [23].

The Lagrange function

$$L(x,u) = f(x) - \sum_{j=1}^{m} u_j g_j(x)$$

is a tool frequently used in optimization. The vector $u = (u_1, \ldots, u_m)$, is the set of Lagrange multipliers [23].

The matrix B_k is defined as the Hessian of a quadratic approximation to the Lagrange function at a given point (x_k, u_k) [23]. In other words,

$$B_k = \nabla_x^2 L(x_k, u_k)$$

This leads to the definition of the quadratic programming subproblem.

$$\begin{aligned} &\text{Minimize} && \tfrac{1}{2} d^T B_k d + \nabla f(x_k)^T d \\ &\text{Subject to:} && \nabla g_j(x_k)^T d + g_j(x_k) = 0, \quad j = 1, \ldots, m_e \end{aligned} \tag{31}$$

$$\nabla g_j(x_k)^T d + g_j(x_k) \geq 0, \quad j = m_e + 1, \ldots, m$$

In essence, (31) is a minimization of the local quadratic approximation to the Lagrangian of the objective function, subject to the linearized constraints of the original problem.

The solution to (31) is a vector $d_k = (d_1, \ldots, d_n)$, found along with $u_k = (u_1, \ldots, u_m)$, the optimal Lagrange multipliers of (31), where k is the iteration number [23]. The direction vector d_k is used to find the new iterate $x_{k+1} = x_k + \alpha_k d_k$ [22].

The *Kuhn-Tucker conditions* for (31):

$$\begin{array}{llll} a) & \nabla_x L(x,u) = 0, & & \\ b) & g_j(x) = 0, & j = 1, \ldots, m_e, & \\ c) & g_j(x) \geq 0, & j = m_e + 1, \ldots, m, & \quad (32) \\ d) & u_j \geq 0, & j = m_e + 1, \ldots, m, & \\ e) & g_j(x) u_j = 0, & j = m_e + 1, \ldots, m, & \end{array}$$

provide the necessary conditions for optimality [21:199]. If the objective function is convex; that is, **H** is positive semi-definite or positive definite and x satisfies all conditions in (32), then $x = x^*$ is the optimal point [20:207].

A.2 The SSQP Algorithm

The basics of SQP are simple and well-known. However, convergence problems have lead investigators to improve the method at the cost of greater complexity.

A.2.1 Step 1: initialization. The user will select x_l, x_u, and the initial values of x. Although not required, a feasible x chosen to be near the optimal point will speed convergence. A judicious choice of x_l and x_u will also shorten runtime and reduce the likelihood of a failure. The final accuracy parameter ϵ is also user selectable and the default value is 10^{-7}.

A.2.2 Step 2: Solve the QP subproblem. Under some conditions, the feasible region of the QP subproblem may be empty while the main problem is feasible [23]. for this reason, the Quadratic Programming subproblem is modified.

$$\begin{array}{lll} \text{Minimize} & \frac{1}{2}d^T B_k d + \nabla f(x_k)^T d + \frac{1}{2}\varrho_k \delta^2 & \\ \text{Subject to:} & \nabla g_j(x_k)^T d + (1-\delta) g_j(x_k) \begin{Bmatrix} = \\ \geq \end{Bmatrix} 0, & j \in J_k, \\ & \nabla g_j(x_{k(j)})^T d + g_j(x_k) \geq 0, & j \in K_k, \end{array} \tag{33}$$

where

$$J_k = \{1,\ldots,m_e\} \cup \{j : m_e < j \leq m, g_j(x_k) \leq \epsilon \text{ or } v_j^{(k)} > 0\},$$

$$K_k = \{1,\ldots,m\} \text{ excluding } J_k$$

[22:488-9]. The additional variable δ will take on values between zero and one. It is added to prevent inconsistency, but the program only assigns delta a nonzero value only if the subroutine solving the QP subproblem reports an error [22:491]. The term ϱ is an additional penalty parameter to control the influence ofδ on the solution [22:489].

The QP subproblem is solved by the subroutine QLD. However, any other available routine from IMSL or NAG could work in its place. QLD solves the QP subproblem by using an algorithm similar to Powell's ZQPCXV [24]. Subroutine QLD works by first finding the unconstrained minimum of the QP objective function. Next, all violated constraints are added to a "Working Set." Then, the QP objective function is minimized again subject to the constraints in the working set. These steps are repeated until no further improvement in the QP objective function can be obtained [21]. NOTE: The working set of constraints may change during this process [21].

A.2.3 Step 3: Finding the New Iterate. The solution vector d_k and the optimal multiplier set u_k of the QP subproblem are found by (31), or (33) as appropriate. The new iterate will be

$$x_{k+1} = x_k + \alpha_k d_k$$

however, it is necessary to update the multiplier estimates v_k for the main problem as well [22:487-8]. This is done at the same time by [21:488]

$$v_{k+1} = v_k + \alpha_k(u_k - v_k).$$

The only quantity left to determine is the step length α_k. The step length is found by minimizing a merit function [22:487]

$$\phi(\alpha) = \psi_{r_k}\left(\begin{pmatrix} x_k \\ v_k \end{pmatrix} + \alpha \begin{pmatrix} d_k \\ u_k - v_k \end{pmatrix}\right). \tag{34}$$

The vector $r_k = (r_1, \ldots, r_m)$ in (34) is a set of m penalty parameters which must be updated in such a way that d_k is a descent direction [24:]. There are two merit functions available in the program. The L1 exact penalty function

$$\psi_{r_k}(x_k, v_k) = f(x_k) + \sum_{j=1}^{m_e} r_j |g_j(x_k)| + \sum_{j=m_e+1}^{m'} r_j |\min(0, g_j(x_k))| \tag{35}$$

where

$m' = m + 2n$

is user-selectable [22:487-8]. However, the augmented Lagrangian function

$$\psi_{r_k}(x_k, v_k) = f(x_k) - \sum_{j=1}^{m_e}(v_j g_j(x_k) - \frac{1}{2} r_j g_j(x_k)^2)$$
$$- \sum_{j=m_e+1}^{m'} \begin{cases} (v_j g_j(x_k) - \frac{1}{2} r_j g_j(x_k)^2) & \text{if } g_j(x_k) \leq v_j/r_j, \\ \frac{1}{2} v_j^2/r_j & \text{otherwise,} \end{cases} \tag{36}$$

is the default merit function [22:488]. In order to perform the line search and minimize α_k first define $\alpha_{k,0} = 1$ and $i = 1, 2, \ldots,$ maxfun, where maxfun is a user-selectable parameter (default = 8) that limits the number of function-calls on the line search [24:203]. Then let i_k be the first index for which

$$\psi_k(\alpha_{k,i}) \leq \psi_k(0) + \mu \alpha_{k,i} \psi_k'(0)$$

holds [21]. The parameter $0 \leq \mu \leq \frac{1}{2}$ is typically set to 0.1 [24:203]. In order to guarantee that $\psi_k'(0) \leq 0$, the penalty parameter r_k must be updated by

$$r_j^{k+1} = \max\left(\sigma_j^{(k)} r_j^{(k)}, \frac{2m(u_j^{(k)} - v_j^{(k)})^2}{(1 - \delta_k) d_k^T B_k d_k}\right), \quad j = 1, \ldots, m, \tag{37}$$

$$r_{k+1} = (r_1^{k+1}, \ldots, r_m^{k+1})^T, \tag{38}$$

$$\text{where: } \sigma_j^{(k)} = min\left(1, \frac{k}{\sqrt{r_j^{(k)}}}\right) \tag{39}$$

[24:201-2] Once the new iterate has been found, the problem is checked for convergence.

A.2.4 Step 4: Checking for Convergence. The use of the penalty parameters in the line search will drive the problem toward an improving and feasible x_k. In order for the algorithm to have converged, all of the conditions in (32) must be met. The feasibility of x_k meets three of the conditions and the other two conditions may be checked by

$$\sum_{j=1}^{m} |u_j^{(k)} g_j(x_k)| \leq \epsilon \tag{40}$$

and (41)

$$||\nabla_x L(x_k, u_k)||^2 \leq \epsilon, \tag{42}$$

However, exactly which stopping criterion is applied by the program was not disclosed in the available literature [24:203]. If the solution is not optimal, then $k = k+1$ and the matrix B_k must be updated to $B_{(k+1)}$ for the next iteration. It is apparent, although not certain that the method of updating B_k in NLPQLD is the BFGS method Such a method is described in [20]. Schittkowski indicates the following in the documentation for NLPQLD:

> The update of the matrix B_k can be performed by standard techniques known from unconstrained optimization. In most cases, the BFGS-method is applied, a numerically simple rank-2 correction starting from the identity or any other positive definite matrix. Only the difference vectors $x_{k+1} - x_k$ and $\nabla_x L(x_{k+1}, u_k) - \nabla_x L(x_k, u_k)$ are required. Under some safeguards it is possible to guarantee that all matrices B_k are positive definite [23].

In any case, a new positive-definite B_k is generated and the next iteration begins with another solution to (31) or (33) as appropriate.

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION VARIAB(5),CONSTR(10)
      DIMENSION GRADOF(5),GRADCO(10,5)
      DIMENSION HESSEM(5,5),RHSIDE(5)
      DIMENSION VECMUL(19),BOULOW(5),BOUUPP(5)
      DIMENSION WORKAR(527),IWORKA(39)
      LOGICAL ACTIVE(40)
      COMMON/CMACHE/EPS100,EPS200,EPS300
      OPEN(10,FILE='EMPAUXI.DAT')
C
C   CALL OF NLP-SUBROUTINE, PROGRAM WRITTEN BY EMP
C

      EPS100=1.D-13
      EPS200=1.D-7
      EPS300=1.D-3
      IOUTST=6
      ACCURA=1.D-7
      MAXITE=80
      MAXFUN=8
      SCABOU=1.D+3
      IPRINT=1
      INFAIL=0
      MODEAL=0
      NOVARI=4
      NOCONS=9
      NOEQCO=0
      NOMMAX=10
      NONMAX=5
      NOMIN2=19
      LEWORK=527
      LEIWOR=39
      LEACTI=40


C     THE INITIAL FUEL FOR THE AIRLIFTER, G

      VARIAB(1)=100.0
      BOULOW(1)=0.0
      BOUUPP(1)=1000.0


C     THE INITIAL FUEL FOR THE TANKER, H

      VARIAB(2)=100.0
```

```
      BOULOW(2)=0.0
      BOUUPP(2)=1000.0

C     THE LONGITUDE OF THE RENDEZVOUS POINT, THETA

      VARIAB(3)=30.0
      BOULOW(3)=-25.0
      BOUUPP(3)=75.0

C     THE LATITUDE OF THE RENDEZVOUS POINT, PHI

      VARIAB(4)=30.0
      BOULOW(4)=0.0
      BOUUPP(4)=89.9

      CALL NLPQL1(NOCONS,NOEQCO,NOMMAX,NOVARI,NONMAX,NOMNN2,VARIAB,
     1 OBJFUN,CONSTR,GRADOF,GRADCO,VECMUL,BOULOW,BOUUPP,HESSEM,
     2 RHSIDE,ACCURA,SCABOU,MAXFUN,MAXITE,IPRINT,MODEAL,IOUTST,
     3 INFAIL,WORKAR,LEWORK,IWORKA,LEIWOR,ACTIVE,LEACTI,.TRUE.,
     4 .TRUE.)
C
C   OUTPUT ON RESULT
C
      WRITE(10,9020) INFAIL,IWORKA(1),IWORKA(2),IWORKA(4)
 9020 FORMAT(1X,4I10)
      DO 9000 I=1,NOVARI
      WRITE(10,9030) VARIAB(I)
      VALMUL=VECMUL(NOCONS+I)
      VALMU1=VECMUL(NOCONS+NOVARI+I)
      IF (VALMU1.GT.VALMUL) VALMUL=VALMU1
 9000 WRITE(10,9030) VALMUL
      WRITE(10,9030) OBJFUN
      SUMMUL=0.D+0
      MNNMUL=NOCONS + NOVARI + NOVARI
      DO 9001 J=1,MNNMUL
 9001 SUMMUL=SUMMUL + DABS(VECMUL(J))
      OBJFUN=0.D+0
      DO 9009 J=1,NOCONS
      GGGGGJ=DABS(CONSTR(J))
      IF (J.GT.NOEQCO.AND.CONSTR(J).GT.0.D+0) GGGGGJ=0.D+0
 9009 OBJFUN=OBJFUN + GGGGGJ
      DO 9002 J=1,9
      WRITE(10,9030) CONSTR(J)
 9002 WRITE(10,9030) VECMUL(J)
      WRITE(10,9030) OBJFUN
      WRITE(10,9030) SUMMUL
 9030 FORMAT(1X,D19.8)
C
C END OF MAIN PROGRAMM
```

```
C
      STOP
      END
      SUBROUTINE NLFUNC(NOCONS,NOEQCO,NOMMAX,NOVARI,OBJFUN,CONSTR,
     1     VARIAB,ACTIVE)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CONSTR(NOMMAX),VARIAB(NOVARI)
      LOGICAL ACTIVE(NOMMAX)
C
C   EVALUATION OF PROBLEM FUNCTIONS
C
      A0 = 36.2829
      A1 = -0.027
      B0 = A0
      B1 = A1
      EWC = 370.0
      EWT = EWC
      V = 0.0
      W = 200.0
      AXCGO = 760.0
      AXTKR = AXCGO
      DTR = 3.14159/180.0
      R = 3404 0
      ALT = 31000.0
      ALT = ALT/6076.12
      G=VARIAB(1)
      H=VARIAB(2)
      TH=VARIAB(3)
      FI=VARIAB(4)
      OLONG = 75.0*DTR
      OLAT = 38*DTR
      DLONG = -28.0*DTR
      DLAT = 30.0*DTR
      TLONG = 66.0*DTR
      TLAT = 18.0*DTR

      GMAX = AXCGO - EWC -W
      HMAX = AXTKR - EWT
      RCMAX = (A0 + A1*(EWC + W + GMAX/2))*GMAX

      RTMAX = (B0 + B1*(EWT +  HMAX/2))*HMAX



      DOR = DSIN(OLAT)*DSIN(FI*DTR)+DCOS(DTR*TH - OLONG)
     /     *DCOS(OLAT)*DCOS(DTR*FI)
      DOR = (R+ALT)*DACOS(DOR)
      DRD = DSIN(DLAT)*DSIN(FI*DTR)+DCOS(DLONG - TH*DTR)
     /     *DCOS(DLAT)*DCOS(DTR*FI)
      DRD = (R+ALT)*DACOS(DRD)
      DBR = DSIN(TLAT)*DSIN(FI*DTR)+DCOS(TH*DTR - TLONG)
```

```
     /     +DCOS(TLAT)*DCOS(DTR*FI)
      DBR = (R+ALT)*DACOS(DBR)
      FCC = G+(A0+A1*(EWC + W))/A1
      FCC = FCC - (((A0+A1*(EWC+W+G))**2-2*A1*DOR)**0.5D0)/A1

      FCCA = GMAX+(A0+A1*(EWC + W))/A1

      FCCA = FCCA - (((A0+A1*(EWC+W+GMAX))**2-2*A1*DOR)**0.5D0)/A1



      FRC = -EWC - W -A0/A1 + (((A0+A1*(EWC+W))**2+2*A1*DRD)**0.5D+0)
     /       /A1
      FRCA= -EWC - W -A0/A1 + (((A0+A1*(EWC+W))**2+2*A1*DOR)**0.5D+0)
     /       /A1


      FCT = H - (((B0+B1*(EWT+H))**2-2*B1*DBR)**0.5D+0)/B1 +
     /    (B0+B1*EWT)/B1

      FCTA=HMAX-(((B0+B1*(EWT+HMAX))**2-2*B1*DBR)**0.5D+0)/B1
     /   + (B0+B1*EWT)/B1


      FRT = -EWT - V - B0/B1 + (((B0+B1*(EWT+V))**2+2*B1*
     /    DBR)**0.5D+0)/B1

      FRTA = -EWT - V - B0/B1 + (((B0+B1*(EWT+V))**2+4*B1*
     /    DBR)**0.5D+0)/B1



      PRINT*,' FCC = ',FCC
      PRINT*,' FRC = ',FRC
      PRINT*,' FCT = ',FCT
      PRINT*,' FRT = ',FRT
      PRINT*,' FCC + FRC - G = ',FCC+FRC-G


      V = FCC + FRC + FCT + FRT
C
      OBJFUN=V


      IF (.NOT.ACTIVE(1)) GOTO 5001

C     DISTANCE FROM ORIGIN TO REFUEL POINT MUST BE LESS THAN
C     THE MAXIMUM RANGE OF THE CARGO AIRCRAFT.
```

```
      G1 = RCMAX - DOR
C
      CONSTR(1)=G1
C
 5001 CONTINUE
      IF (.NOT.ACTIVE(2)) GOTO 5002

C     THE DISTANCE FROM THE REFUELING POINT TO THE DESTINATION

C     MUST BE LESS THAN THE MAXIMUM RANGE OF THE CARGO A/C

      G2 = RCMAX - DRD
C
      CONSTR(2)=G2
C
 5002 CONTINUE
      IF (.NOT.ACTIVE(3)) GOTO 5003

C     THE DISTANCE FROM THE TANKER BASE TO THE REFUELING POINT
C     MUST BE LESS THAN 1/2 OF THE TANKER'S RANGE.


      G3 = RTMAX/2 - DBR
C
      CONSTR(3)=G3
C
 5003 CONTINUE
      IF (.NOT.ACTIVE(4)) GOTO 5004

C     THE INITIAL FUEL G MUST BE AT LEAST ENOUGH TO GET THE
C     CARGO A/C TO THE REFUELING POINT.


      G4 = G - FRCA
C
      CONSTR(4)=G4
C
 5004 CONTINUE
      IF (.NOT.ACTIVE(5)) GOTO 5005

C     THE INITIAL FUEL MUST BE LESS THAN THE MAXIMUM AMOUNT THE
C     CARGO A/C CAN CARRY.

      G5 = GMAX - G
C
      CONSTR(5)=G5
C
 5005 CONTINUE

      IF (.NOT.ACTIVE(6)) GOTO 5006
```

```
C     THE TANKER'S INITIAL FUEL MUST BE AT LEAST ENOUGH TO MAKE IT
C     TO THE REFUELING POINT. THIS CONSTRAINT WILL NEVER BE BINDING
C     SO IT WILL BE REPLACED WITH A QUICK EQUATION THAT WILL GIVE THE
C     AMOUNT OF FUEL TRANSFERRED TO THE AIRLIFTER

C     G6 = H - FRTA
C
      G6 = FCC + FRC - G

      CONSTR(6)=G6
C
 5006 CONTINUE


      IF (.NOT.ACTIVE(7)) GOTO 5007

C     THE TANKER'S INITIAL FUEL CANNOT BE MORE THAN ITS MAXIMUM T/O
C     FUEL.

      G7 = HMAX - H
C
      CONSTR(7)=G7
C
 5007 CONTINUE
      IF (.NOT.ACTIVE(8)) GOTO 5008

C     THE SUM OF H AND G IS ALWAYS GREATER THAN OR EQUAL TO THE
C     TOTAL FUEL REQUIRED FOR THE MISSION.

      G8= H + G - FRC - FCC - FRT - FCT
C
      CONSTR(8)=G8
C
 5008 CONTINUE
      IF (.NOT.ACTIVE(9)) GOTO 5009

C     THIS CONSTRAINT IS SUPPOSED TO FORCE THE TANKER TO BRING
C     ENOUGH FUEL FOR THE TRANSPORT AS WELL.

      G9 = GMAX + HMAX - FCCA - FCTA - FRC - FRT
C
      CONSTR(9)=G9
C
 5009 CONTINUE
C
C   END OF NLFUNC
C
      RETURN
      END
      SUBROUTINE NLGRAD(NOCONS,NOEQCO,NOMMAX,NOVARI,OBJFUN,
     1    CONSTR,GRADOF,GRADCO,VARIAB,ACTIVE,CONEPS)
```

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CONSTR(NOMMAX),GRADOF(NOVARI),GRADCO(NOMMAX,NOVARI),
     1    VARIAB(NOVARI),CONEPS(NOMMAX)
      LOGICAL ACTIVE(NOMMAX)
C
C   EVALUATION OF GRADIENTS
C
      ON=1.D+0
      EPS=1.D-7
      DO1 I=1,NOVARI
      XEPS=EPS*DMAX1(ON,DABS(VARIAB(I)))
      XEPSI=ON/XEPS
      VARIAB(I)=VARIAB(I) + XEPS
      CALL NLFUNC(NOCONS,NOEQCO,NOMMAX,NOVARI,FEPS,CONEPS,VARIAB,
     1     ACTIVE)
      GRADOF(I)=(FEPS - OBJFUN)*XEPSI
      DO2 J=1,NOCONS
      IF (.NOT.ACTIVE(J)) GOTO 2
      GRADCO(J,I)=(CONEPS(J) - CONSTR(J))*XEPSI
    2 CONTINUE
    1 VARIAB(I)=VARIAB(I) - XEPS
C
C   END OF NLGRAD
C
      RETURN
      END
```

```
C*********************************************************************
C WHAT FOLLOWS IS THE RAW DATA OBTAINED BY RUNNING THE DIFFERENT
C VERSIONS OF THE MODEL. IF THE CODE IS BEING RECREATED, DO NOT CODE
C THE FOLLOWING MATERIAL:


 FCC =     81.640117766392
 FRC =     189.99999620667
 FCT =     83.189664428001
 FRT =     62.924586847970
 FCC + FRC - G =     156.49990393841


     * FINAL CONVERGENCE ANALYSIS

        OBJECTIVE FUNCTION VALUE:  F(X) =  0.41775436D+03
        APPROXIMATION OF SOLUTION:  X =
           0.11514021D+03  0.30261415D+03  0.67724366D+02  0.35266921D+02
        APPROXIMATION OF MULTIPLIERS:  U =
           0.00000000D+00  0.72584446D-01  0.00000000D+00  0.00000000D+00
           0.00000000D+00  0.00000000D+00  0.00000000D+00  0.12394133D+00
           0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
           0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
           0.00000000D+00
        CONSTRAINT VALUES:  G(X) =
           0.19404248D+04 -0.34229743D-07  0.24994240D+04  0.37440141D+02
           0.74859790D+02  0.15649991D+03  0.87385849D+02 -0.46004232D-06
           0.14231275D+03
        DISTANCE FROM LOWER BOUND:  XL-X =
          -0.11514021D+03 -0.30261415D+03 -0.67724366D+02 -0.35266921D+02
        DISTANCE FROM UPPER BOUND:  XU-X =
           0.88485979D+03  0.69738585D+03  0.32275634D+02  0.54633079D+02
        NUMBER OF FUNC-CALLS:  NFUNC =  18
        NUMBER OF GRAD-CALLS:  NGRAD =  17
        NUMBER OF QL-CALLS:    NQL   =  17
```

```
C   CLAYTON PFLIEGER/DR SCHITTKOWSKY DEC 1992
C
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION VARIAB(5),CONSTR(10)
      DIMENSION GRADOF(5),GRADCO(10,5)
      DIMENSION HESSEM(5,5),RHSIDE(5)
      DIMENSION VECMUL(19),BOULOW(5),BOUUPP(5)
      DIMENSION WORKAR(527),IWORKA(39)
      LOGICAL ACTIVE(40)
      COMMON/CMACHE/EPS100,EPS200,EPS300
      OPEN(10,FILE='EMPAUXI.DAT')


      EPS100=1.D-13
      EPS200=1.D-7
      EPS300=1.D-3
      IOUTST=6
      ACCURA=1.D-7
      MAXITE=80
      MAXFUN=16
      SCABOU=1.D+3
      IPRINT=1
      INFAIL=0
      MODEAL=0
      NOVARI=4
      NOCONS=9
      NOEQCO=0
      NOMMAX=10
      NONMAX=5
      NOMNN2=19
      LEWORK=527
      LEIWOR=39
      LEACTI=40


C     THIS SETS THE UPPER BOUND, THE LOWER BOUND, AND THE
C     INITIAL GUESS OF THE DECISION VARIABLES.

C     THE AIRLIFTER INITIAL FUEL, G

      VARIAB(1)= 00.0
      BOULOW(1)= 0.0
      BOUUPP(1)= 1000.0
```

```
C     THE TANKER INITIAL FUEL, H

      VARIAB(2)= 00.0
      BOULOW(2)= 0.0
      BOUUPP(2)= 1000.0

C     THE LONGITUDE OF THE RENDEZVOUS POINT, THETA

      VARIAB(3)=  15.0
      BOULOW(3)= -13.0
      BOUUPP(3)=  75.0

C     THE LATITUDE OF THE RENDEZVOUS POINT, PHI

      VARIAB(4)= 40.0
      BOULOW(4)= 35.0
      BOUUPP(4)= 89.9

      CALL NLPQL1(NOCONS,NOEQCO,NOMMAX,NOVARI,NONMAX,NOMNN2,VARIAB,
     1  OBJFUN,CONSTR,GRADOF,GRADCO,VECMUL,BOULOW,BOUUPP,HESSEM,
     2  RHSIDE,ACCURA,SCABOU,MAXFUN,MAXITE,IPRINT,MODEAL,IOUTST,
     3  INFAIL,WORKAR,LEWORK,IWORKA,LEIWOR,ACTIVE,LEACTI,.TRUE.,
     4  .TRUE.)
C
C   OUTPUT ON RESULT
C
      WRITE(10,9020) INFAIL,IWORKA(1),IWORKA(2),IWORKA(4)
 9020 FORMAT(1X,4I10)
      DO 9000 I=1,NOVARI
      WRITE(10,9030) VARIAB(I)
      VALMUL=VECMUL(NOCONS+I)
      VALMU1=VECMUL(NOCONS+NOVARI+I)
      IF (VALMU1.GT.VALMUL) VALMUL=VALMU1
 9000 WRITE(10,9030) VALMUL
      WRITE(10,9030) OBJFUN
      SUMMUL=0.D+0
      MNNMUL=NOCONS + NOVARI + NOVARI
      DO 9001 J=1,MNNMUL
 9001 SUMMUL=SUMMUL + DABS(VECMUL(J))
      OBJFUN=0.D+0
      DO 9009 J=1,NOCONS
      GGGGGJ=DABS(CONSTR(J))
      IF (J.GT.NOEQCO.AND.CONSTR(J).GT.0.D+0) GGGGGJ=0.D+0
 9009 OBJFUN=OBJFUN + GGGGGJ
      DO 9002 J=1,9
      WRITE(10,9030) CONSTR(J)
 9002 WRITE(10,9030) VECMUL(J)
      WRITE(10,9030) OBJFUN
      WRITE(10,9030) SUMMUL
 9030 FORMAT(1X,D19.8)
      STOP
```

```
      END
C
C END OF MAIN PROGRAMM
C

      SUBROUTINE NLFUNC(NOCONS,NOEQCO,NOMMAX,NOVARI,OBJFUN,CONSTR,
     1     VARIAB,ACTIVE)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CONSTR(NOMMAX),VARIAB(NOVARI)
      LOGICAL ACTIVE(NOMMAX)


      A0    =  45.9127
      A1    =  -0.0531
      B0    =  74.9700
      B1    =  -0.1353
      EWC   = 152.685
      EWT   = 100.0
      V     =   0.0
      W     =  50.0
      AXCGO = 323.1
      AXTKR = 300.0


      DTR   = 3.141592654/180.0

      OLONG = -12.6*DTR
      OLAT  =  46.03*DTR
      DLONG =  74.6*DTR
      DLAT  =  40.02*DTR
      TLONG =  27.1*DTR
      TLAT  =  38.7*DTR

      R     = 3443.92
      ALT   = 31000.0
      ALT   = ALT/6076.12

      G     = VARIAB(1)
      H     = VARIAB(2)
      TH    = VARIAB(3)
      FI    = VARIAB(4)

C     GMAX AND HMAX ARE THE MAXIMUM INITIAL FUEL LOADS OF THE
C     CARGO A/C AND THE TANKER RESPECTIVELY.

      GMAX = AXCGO - EWC - W
      HMAX = AXTKR - EWT

C     R*MAX IS THE MAXIMUM RANGE OF THE * A/C
```

```
      RCMAX = (A0 + A1*(EWC + W + GMAX/2))*GMAX
      RTMAX = (B0 + B1*(EWT +  HMAX/2))*HMAX

C     DOR IS THE GREAT CIRCLE DISTANCE FROM THE ORIGIN TO
C     THE REFUELING POINT

      DOR = DSIN(OLAT)*DSIN(FI*DTR)+DCOS(DTR*TH - OLONG)
     /      *DCOS(OLAT)*DCOS(DTR*FI)
      DOR = (R+ALT)*DACOS(DOR)

C     DRD IS THE GREAT CIRCLE DISTANCE FROM THE REFUELING
C     POINT TO THE DESTINATION OF THE CARGO A/C

      DRD = DSIN(DLAT)*DSIN(FI*DTR)+DCOS(DLONG - TH*DTR)
     /      *DCOS(DLAT)*DCOS(DTR*FI)
      DRD = (R+ALT)*DACOS(DRD)

C     DBR IS THE GREAT CIRCLE DISTANCE FROM THE TANKER BASE
C     TO THE REFUELING POINT. BECAUSE OF THE ASSUMED SYMMETRY,
C     THERE IS NO DRB. IT IS ASSUMED TO BE EQUAL TO DBR.

      DBR = DSIN(TLAT)*DSIN(FI*DTR)+DCOS(TH*DTR - TLONG)
     /      *DCOS(TLAT)*DCOS(DTR*FI)
      DBR = (R+ALT)*DACOS(DBR)

C     FCC IS THE FUEL CONSUMED BY THE CARGO AIRCRAFT IN REACHING
C     THE REFUELING POINT.

      FCC = G + (A0 + A1*(EWC + W))/A1
      FCC = FCC - (((A0+A1*(EWC+W+G))**2-2*A1*DOR)**0.5D0)/A1

C     FCCA IS THE FUEL CONSUMED BY THE CARGO AIRCRAFT IF IT WERE
C     TO FLY FROM ITS ORIGIN TO THE REFUELING POINT WITH FULL
C     TANKS

      FCCA = GMAX + (A0 + A1*(EWC + W))/A1
      FCCA = FCCA - (((A0+A1*(EWC+W+GMAX))**2-2*A1*DOR)**0.5D0)/A1

C     FRC IS THE FUEL REQUIRED BY THE CARGO A/C TO FLY FROM THE
C     REFUELING POINT TO THE DESTINATION.

      FRC=-EWC-W-A0/A1+(((A0+A1*(EWC+W))**2+2*A1*DRD)**0.5D+0)/A1

C     FRCA IS THE MINIMUM FUEL REQUIRED BY THE CARGO A/C TO FLY FROM
C     THE ORIGIN TO THE REFUELING POINT. FRCA .LE. FCC

      FRCA=-EWC-W-A0/A1+(((A0+A1*(EWC+W))**2+2*A1*DOR)**0.5D+0)/A1

C     FCT IS THE FUEL CONSUMED BY THE TANKER IN REACHING THE RE-
```

```
C     FUELING POINT

      FCT = H - [illegible]+B1*(EWT+H))**2-2*B1*DBR)**0.5D+0)/B1 +
     /    (B0 + B1*EWT)/B1

C     FCTA IS THE AMOUNT OF FUEL THE TANKER WOULD CONSUME IN FLYING
C     TO THE REFUELING POINT IF IT WERE TO TAKE OFF WITH FULL TANKS.

      FCTA=HMAX-(((B0+B1*(EWT+HMAX))**2-2*B1*DBR)**0.5D+0)/B1
     /   + (B0 + B1*EWT)/B1

C     FRT IS THE FUEL REQUIRED BY THE TANKER TO MAKE IT HOME FROM
C     THE REFUELING POINT.

      FRT = -EWT - V - B0/B1 + (((B0+B1*(EWT+V))**2+2*B1*
     /    DBR)**0.5D+0)/B1

C     FRTA IS THE MINIMUM AMOUNT OF FUEL REQUIRED BY THE TANKER
C     TO FLY TWICE THE DISTANCE TO THE REFUELING POINT.

      FRTA = -EWT - V - B0/B1 + (((B0+B1*(EWT+V))**2+4*B1*
     /    DBR)**0.5D+0)/B1

      PRINT*,' FCC = ',FCC
      PRINT*,' FRC = ',FRC
      PRINT*,' FCT = ',FCT
      PRINT*,' FRT = ',FRT
      PRINT*,' FCC + FRC - G = ',FCC+FRC-G



C     V IS THE OBJECTIVE FUNCTION THAT WE SEEK TO MINIMIZE.
C     IT IS EQUAL TO THE SUM OF THE FUELS NECESSARY TO FLY
C     THE MISSION.

      V = FCC + FRC + FCT + FRT
      OBJFUN = V


C     THE FOLLOWING IS THE SET OF CONSTRAINTS FOR THIS NLP


C     DISTANCE FROM ORIGIN TO REFUEL POINT MUST BE LESS THAN
C     THE MAXIMUM RANGE OF THE CARGO AIRCRAFT.

      IF (.NOT.ACTIVE(1)) GOTO 5001

      G1 = RCMAX - DOR
      CONSTR(1) = G1
```

```
 5001 CONTINUE

C     THE DISTANCE FROM THE REFUELING POINT TO THE DESTINATION
C     MUST BE LESS THAN THE MAXIMUM RANGE OF THE CARGO A/C

      IF (.NOT.ACTIVE(2)) GOTO 5002

      G2 = RCMAX - DRD
      CONSTR(2) = G2

 5002 CONTINUE

C     THE DISTANCE FROM THE TANKER BASE TO THE REFUELING POINT
C     MUST BE LESS THAN 1/2 OF THE TANKER'S RANGE.

      IF (.NOT.ACTIVE(3)) GOTO 5003

      G3 = RTMAX/2 - DBR
      CONSTR(3) = G3

 5003 CONTINUE

C     THE INITIAL FUEL G MUST BE AT LEAST ENOUGH TO GET THE
C     CARGO A/C TO THE REFUELING POINT.

      IF (.NOT.ACTIVE(4)) GOTO 5004

      G4 = G - FRCA
      CONSTR(4) = G4

 5004 CONTINUE

C     THE INITIAL FUEL MUST BE LESS THAN THE MAXIMUM AMOUNT THE
C     CARGO A/C CAN CARRY.

      IF (.NOT.ACTIVE(5)) GOTO 5005

      G5 = GMAX - G
      CONSTR(5) = G5

 5005 CONTINUE

C     THE TANKER'S INITIAL FUEL MUST BE AT LEAST ENOUGH TO MAKE IT
C     TO THE REFUELING POINT. THIS CONSTRAINT WILL NEVER BE BINDING
C     SO IT WILL BE REPLACED WITH A QUICK EQUATION THAT WILL GIVE THE
C     AMOUNT OF FUEL TRANSFERRED TO THE AIRLIFTER
```

```
      IF (.NOT.ACTIVE(6)) GOTO 5006

C     G6 = H - FRTA
C
      G6 = FCC + FRC - G

      CONSTR(6)=G6

 5006 CONTINUE

C     THE TANKER'S INITIAL FUEL CANNOT BE MORE THAN ITS MAXIMUM T/O
C     FUEL.

      IF (.NOT.ACTIVE(7)) GOTO 5007

      G7 = HMAX - H
      CONSTR(7)=G7

 5007 CONTINUE


C     THE SUM OF H AND G IS ALWAYS GREATER THAN OR EQUAL TO THE
C     TOTAL FUEL REQUIRED FOR THE MISSION.

      IF (.NOT.ACTIVE(8)) GOTO 5008

      G8 = H + G - FRC - FCC - FRT - FCT
      CONSTR(8) = G8

 5008 CONTINUE

C     THIS CONSTRAINT IS SUPPOSED TO FORCE THE TANKER TO BRING
C     ENOUGH FUEL FOR THE TRANSPORT AS WELL.

      IF (.NOT.ACTIVE(9)) GOTO 5009

      G9 = GMAX + HMAX - FCCA - FCTA - FRC - FRT
      CONSTR(9)=G9

 5009 CONTINUE

      RETURN
      END
C
C   END OF NLFUNC
C

      SUBROUTINE NLGRAD(NOCONS,NOEQCO,NOMMAX,NOVARI,OBJFUN,
     1    CONSTR,GRADOF,GRADCO,VARIAB,ACTIVE,CONEPS)
```

```
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CONSTR(NOMMAX),GRADOF(NOVARI),GRADCO(NOMMAX,NOVARI),
     1    VARIAB(NOVARI),CONEPS(NOMMAX)
      LOGICAL ACTIVE(NOMMAX)
C
C   EVALUATION OF GRADIENTS
C
      ON=1.D+0
      EPS=1.D-7
      DO1 I=1,NOVARI
      XEPS=EPS*DMAX1(ON,DABS(VARIAB(I)))
      XEPSI=ON/XEPS
      VARIAB(I)=VARIAB(I) + XEPS
      CALL NLFUNC(NOCONS,NOEQCO,NOMMAX,NOVARI,FEPS,CONEPS,VARIAB,
     1     ACTIVE)
      GRADOF(I)=(FEPS - OBJFUN)*XEPSI
      DO2 J=1,NOCONS
      IF (.NOT.ACTIVE(J)) GOTO 2
      GRADCO(J,I)=(CONEPS(J) - CONSTR(J))*XEPSI
    2 CONTINUE
    1 VARIAB(I)=VARIAB(I) - XEPS
C
C   END OF NLGRAD
C
      RETURN
      END
```

```
C*********************************************************************
C WHAT FOLLOWS IS THE RAW DATA OBTAINED BY RUNNING THE DIFFERENT
C VERSIONS OF THE MODEL. IF THE CODE IS BEING RECREATED, DO NOT CODE
C THE FOLLOWING MATERIAL:


Optimal solution to model2.f:

 FCC =     53.359618620320
 FRC =     65.411663298500
 FCT =     2.7643093289953D-06
 FRT =     2.3659734438297D-06
 FCC + FRC - G =     65.411663298502


     * FINAL CONVERGENCE ANALYSIS

         OBJECTIVE FUNCTION VALUE:  F(X) =  0.11877129D+03
         APPROXIMATION OF SOLUTION:  X =
            0.53359619D+02  0.65435935D+02  0.27100003D+02  0.38700001D+02
         APPROXIMATION OF MULTIPLIERS:  U =
            0.00000000D+00  0.00000000D+00  0.00000000D+00  0.80607639D-01
            0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
            0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
            0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
            0.00000000D+00
         CONSTRAINT VALUES:  G(X) =
            0.20476306D+04  0.16620044D+04  0.47909999D+04 -0.26005864D-11
            0.67055381D+02  0.65411663D+02  0.13456407D+03  0.24266155D-01
            0.19565933D+03
         DISTANCE FROM LOWER BOUND:  XL-X =
           -0.53359619D+02 -0.65435935D+02 -0.40100003D+02 -0.37000010D+01
         DISTANCE FROM UPPER BOUND:  XU-X =
            0.94664038D+03  0.93456407D+03  0.47899997D+02  0.51199999D+02
         NUMBER OF FUNC-CALLS:  NFUNC = 144
         NUMBER OF GRAD-CALLS:  NGRAD =  72
         NUMBER OF QL-CALLS:    NQL   =  72
```

Appendix D. Model 3 code

```
C   CLAYTON PFLIEGER/DR SCHITTKOWSKY DEC 1992
C   THIS IS MODEL3.F
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION VARIAB(5),CONSTR(10)
      DIMENSION GRADOF(5),GRADCO(10,5)
      DIMENSION HESSEM(5,5),RHSIDE(5)
      DIMENSION VECMUL(19),BOULOW(5),BOUUPP(5)
      DIMENSION WORKAR(527),IWORKA(39)
      LOGICAL ACTIVE(40)
      COMMON/CMACHE/EPS100,EPS200,EPS300
      OPEN(10,FILE='EMPAUXI.DAT')

C        1         2         3         4         5         6         7
C2345678901234567890123456789012345678901234567890123456789012345678901


      EPS100=1.D-13
      EPS200=1.D-7
      EPS300=1.D-3
      IOUTST=6
      ACCURA=1.D-7
      MAXITE=80
      MAXFUN=16
      SCABCU=1.D+3
      IPRINT=1
      INFAIL=0
      MODEAL=0
      NOVARI=4
      NOCONS=9
      NOEQCO=0
      NOMMAX=10
      NONMAX=5
      NOMNN2=19
      LEWORK=527
      LEIWOR=39
      LEACTI=40

      EWT = 100.0
      AXTKR = 300.0
      HMAX = AXTKR - EWT

C     THIS SETS THE UPPER BOUND, THE LOWER BOUND, AND THE
```

```
C     INITIAL GUESS OF THE DECISION VARIABLES.

      VARIAB(1) =    0.0
      BOULOW(1) =    0.0
      BOUUPP(1) = 1000.0

      VARIAB(2) =    0.0
      BOULOW(2) =    0.0
      BOUUPP(2) =   HMAX

      VARIAB(3) =   15.0
      BOULOW(3) =  -13.0
      BOUUPP(3) =   75.0

      VARIAB(4) =   40.0
      BOULOW(4) =   35.0
      BOUUPP(4) =   89.0

      CALL NLPQL1(NOCONS,NOEQCO,NOMMAX,NOVARI,NONMAX,NOMNN2,VARIAB,
     1  OBJFUN,CONSTR,GRADOF,GRADCO,VECMUL,BOULOW,BOUUPP,HESSEM,
     2  RHSIDE,ACCURA,SCABOU,MAXFUN,MAXITE,IPRINT,MODEAL,IOUTST,
     3  INFAIL,WORKAR,LEWORK,IWORKA,LEIWOR,ACTIVE,LEACTI,.FALSE.,
     4  .TRUE.)
C
C   OUTPUT ON RESULT
C
      WRITE(10,9020) INFAIL,IWORKA(1),IWORKA(2),IWORKA(4)
 9020 FORMAT(1X,4I10)
      DO 9000 I=1,NOVARI
      WRITE(10,9030) VARIAB(I)
      VALMUL=VECMUL(NOCONS+I)
      VALMU1=VECMUL(NOCONS+NOVARI+I)
      IF (VALMU1.GT.VALMUL) VALMUL=VALMU1
 9000 WRITE(10,9030) VALMUL
      WRITE(10,9030) OBJFUN
      SUMMUL=0.D+0
      MNNMUL=NOCONS + NOVARI + NOVARI
      DO 9001 J=1,MNNMUL
 9001 SUMMUL=SUMMUL + DABS(VECMUL(J))
      OBJFUN=0.D+0
      DO 9009 J=1,NOCONS
      GGGGGJ=DABS(CONSTR(J))
      IF (J.GT.NOEQCO.AND.CCNSTR(J).GT.0.D+0) GGGGGJ=0.D+0
 9009 OBJFUN=OBJFUN + GGGGGJ
      DO 9002 J=1,9
      WRITE(10,9030) CONSTR(J)
 9002 WRITE(10,9030) VECMUL(J)
      WRITE(10,9030) OBJFUN
      WRITE(10,9030) SUMMUL
 9030 FORMAT(1X,D19.8)
      STOP
```

```
      END
C
C END OF MAIN PROGRAMM
C

      SUBROUTINE NLFUNC(NOCONS,NOEQCO,NOMMAX,NOVARI,OBJFUN,CONSTR,
     1     VARIAB,ACTIVE)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CONSTR(NOMMAX),VARIAB(NOVARI)
      LOGICAL ACTIVE(NOMMAX)

C     THESE ARE AIRCRAFT PERFORMANCE CONSTANTS.

      A0 = 45.9127
      A1 = -0.0531
      B0 = 74.9700
      B1 = -0.1353
      C1 = 0.015
      C0 = -0.08
      D1 = 0.016
      D0 = -0.127

      EWC   = 152.685
      EWT   = 100.0
      V     =   0.0
      W     =  50.0
      AXCGO = 323.1
      AXTKR = 300.0

C     GMAX AND HMAX ARE THE MAXIMUM INITIAL FUEL LOADS OF THE
C     CARGO A/C AND THE TANKER RESPECTIVELY.

      GMAX = AXCGO - EWC - W
      HMAX = AXTKR - EWT


C     R*MAX IS THE MAXIMUM RANGE OF THE * A/C

      RCMAX = (A0 + A1*(EWC + W + GMAX/2))*GMAX
      RTMAX = (B0 + B1*(EWT +  HMAX/2))*HMAX


      DESC1 = 1.2
      DESC2 = 1.2
      PAL1  = 1.3
      PAL2  = 1.0
      RES1  = 6.7
      RES2  = 3.0
```

```
      DTR = 3.141592654/180.0

      OLONG = -12.6  *DTR
      OLAT  =  46.03 *DTR

      DLONG =  74.6  *DTR
      DLAT  =  40.02 *DTR

      TLONG =  27.1  *DTR
      TLAT  =  38.7  *DTR

      ALONG = -12.6  *DTR
      ALAT  =  46.03 *DTR

      R = 3443.92
      ALT = 31000.0
      ALT   = ALT/6076.12


      G=VARIAB(1)
      H=VARIAB(2)
      TH=VARIAB(3)
      FI=VARIAB(4)


C     DOR IS THE GREAT CIRCLE DISTANCE FROM THE ORIGIN TO
C     THE REFUELING POINT. THE CONDITION THAT DOR BE AT LEAST 100
C     NAUTICAL MILES IS INCLUDED.

      DOR = DSIN(OLAT)*DSIN(FI*DTR)+DCOS(DTR*TH - OLONG)
     /     *DCOS(OLAT)*DCOS(DTR*FI)
      DOR = (R+ALT)*DACOS(DOR)

      IF (DOR .GT. 100.0) GOTO 100

      DOR = 100.0

 100  CONTINUE


C     DRD IS THE GREAT CIRCLE DISTANCE FROM THE REFUELING
C     POINT TO THE DESTINATION OF THE CARGO A/C

      DRD = DSIN(DLAT)*DSIN(FI*DTR)+DCOS(DLONG - TH*DTR)
     /     *DCOS(DLAT)*DCOS(DTR*FI)
      DRD = (R+ALT)*DACOS(DRD)

      IF (DRD .GT. 100.0) GOTO 101
```

```
      DRD = 100.0
  101 CONTINUE

C     DBR IS THE GREAT CIRCLE DISTANCE FROM THE TANKER BASE
C     TO THE REFUELING POINT.

      DBR = DSIN(TLAT)*DSIN(FI*DTR)+DCOS(TH*DTR - TLONG)
     /      *DCOS(TLAT)*DCOS(DTR*FI)
      DBR = (R+ALT)*DACOS(DBR)

      IF (DBR .GT. 100.0) GOTO 102

      DBR = 100.0

  102 CONTINUE

C     DBR IS THE GREAT CIRCLE DISTANCE FROM THE REFUELING POINT TO
C     THE TANKER BASE.

      DRB = DSIN(TLAT)*DSIN(FI*DTR)+DCOS(TLONG - TH*DTR)
     /      *DCOS(TLAT)*DCOS(DTR*FI)
      DRB = (R+ALT)*DACOS(DRB)

      IF (DRB .GT. 100.0) GOTO 103

      DRB = 100.0

  103 CONTINUE

C     DRA IS THE GREAT CIRCLE DISTANCE FROM THE REFUELING POINT TO
C     THE ALTERNATE.

      DRA = DSIN(ALAT)*DSIN(FI*DTR)+DCOS(ALONG - TH*DTR)
     /      *DCOS(ALAT)*DCOS(DTR*FI)
      DRA = (R+ALT)*DACOS(DRA)

      IF (DRA .GT. 100.0) GOTO 104

      DRA = 100.0

  104 CONTINUE

C     FCC IS THE FUEL CONSUMED BY THE CARGO AIRCRAFT IN REACHING
C     THE REFUELING POINT.
```

```
      FCC = G + (A0 + A1*(EWC + W))/A1
      FCC = FCC - (((A0+A1*(EWC+W+G))**2-2*A1*DOR)**0.5D0)/A1
      FCC = FCC + C1*(EWC + W + G) + C0

C     FCCA IS THE FUEL CONSUMED BY THE CARGO AIRCRAFT IF IT WERE
C     TO FLY FROM ITS ORIGIN TO THE REFUELING POINT WITH FULL
C     TANKS

      FCCA = GMAX + (A0 + A1*(EWC + W))/A1
      FCCA = FCCA - (((A0+A1*(EWC+W+GMAX))**2-2*A1*DOR)**0.5D0)/A1
      FCCA = FCCA + C1*(EWC + W + GMAX) + C0

C     FRC IS THE FUEL REQUIRED BY THE CARGO A/C TO FLY FROM THE
C     REFUELING POINT TO THE DESTINATION.

      DRDA = DRD - 100.0
      FRC=-EWC-W-A0/A1+(((A0+A1*(EWC+W))**2+2*A1*DRDA)**0.5D+0)/A1
      FRC = FRC + DESC1 + PAL1 + RES1

C     FRD IS THE FUEL REQUIRED TO DIVERT TO THE ALTERNATE FROM
C     THE REFUELING POINT IF THE CARGO A/C IS NOT REFUELED

      DRAA = DRA - 100.0
      FRD=-EWC-W-A0/A1+(((A0+A1*(EWC+W))**2+2*A1*DRAA)**0.5D+0)/A1
      FRD = FRD + DESC1 + PAL1 + RES1

C     FRCA IS THE MINIMUM FUEL REQUIRED BY THE CARGO A/C TO FLY FROM
C     THE ORIGIN TO THE REFUELING POINT. FRCA .LE. FCC

      DORA = DOR - 100.0
      FRCA=-EWC-W-A0/A1+(((A0+A1*(EWC+W))**2+2*A1*DORA)**0.5D+0)/A1
      FRCA = FRCA + DESC1 + PAL1 + RES1

C     FCT IS THE FUEL CONSUMED BY THE TANKER IN REACHING THE RE-
C     FUELING POINT.

      FCT = H - (((B0+B1*(EWT+H))**2-2*B1*DBR)**0.5D+0)/B1 +
     /    (B0 + B1*EWT)/B1
      FCT = FCT + D1*(EWT + H) + D0

C     FCTA IS THE AMOUNT OF FUEL THE TANKER WOULD CONSUME IN FLYING
C     TO THE REFUELING POINT IF IT WERE TO TAKE OFF WITH FULL TANKS.

      FCTA=HMAX-(((B0+B1*(EWT+HMAX))**2-2*B1*DBR)**0.5D+0)/B1
```

```
     /   + (B0 + B1*EWT)/B1
      FCTA = FCTA + D1*(EWT + HMAX) + D0


C     FRT IS THE FUEL REQUIRED BY THE TANKER TO MAKE IT HOME FROM
C     THE REFUELING POINT.

      DRBA = DRB - 100.0
      FRT = -EWT - V - B0/B1 + (((B0+B1*(EWT+V))**2+2*B1*
     /    DRBA)**0.5D+0)/B1
      FRT = FRT + DESC2 + PAL2 + RES2


C     FRTA IS THE MINIMUM AMOUNT OF FUEL REQUIRED BY THE TANKER
C     TO FLY TWICE THE DISTANCE TO THE REFUELING POINT.

      FRTA = -EWT - V - B0/B1 + (((B0+B1*(EWT+V))**2+4*B1*
     /    DBR)**0.5D+0)/B1


C     WFL IS THE AMOUNT OF FUEL THAT THE CARGO A/C WILL HAVE LEFT
C     AT THE REFUELING POINT.

      WFL = G - FCC



      PRINT*,' FCC = ',FCC
      PRINT*,' FRC = ',FRC
      PRINT*,' FCT = ',FCT
      PRINT*,' FRT = ',FRT
      PRINT*,' FCC + FRC - G = ',FCC+FRC-G


C     V IS THE OBJECTIVE FUNCTION THAT WE SEEK TO MINIMIZE.
C     IT IS EQUAL TO THE SUM OF THE FUELS NECESSARY TO FLY
C     THE MISSION.

      V = FCC + FRC + FCT + FRT
      OBJFUN = V



C     THE FOLLOWING IS THE SET OF CONSTRAINTS FOR THIS NLP



C     DISTANCE FROM ORIGIN TO REFUEL POINT MUST BE LESS THAN
C     THE MAXIMUM RANGE OF THE CARGO AIRCRAFT.

      IF (.NOT.ACTIVE(1)) GOTO 5001
```

```
      G1 = GMAX - FCC
      CONSTR(1) = G1

 5001 CONTINUE

C     THE DISTANCE FROM THE REFUELING POINT TO THE DESTINATION
C     MUST BE LESS THAN THE MAXIMUM RANGE OF THE CARGO A/C

      IF (.NOT.ACTIVE(2)) GOTO 5002

      G2 = GMAX - FRC
      CONSTR(2) = G2

 5002 CONTINUE

C     THE DISTANCE FROM THE TANKER BASE TO THE REFUELING POINT
C     MUST BE LESS THAN THE TANKER'S OUT AND BACK RANGE.

      IF (.NOT.ACTIVE(3)) GOTO 5003

      G3 = HMAX - (FCT + FRT)
      CONSTR(3) = G3

 5003 CONTINUE

C     THE INITIAL FUEL G MUST BE AT LEAST ENOUGH TO GET THE
C     CARGO A/C TO THE REFUELING POINT.

      IF (.NOT.ACTIVE(4)) GOTO 5004

      G4 = G - FRCA
      CONSTR(4) = G4

 5004 CONTINUE

C     THE INITIAL FUEL MUST BE LESS THAN THE MAXIMUM AMOUNT THE
C     CARGO A/C CAN CARRY.

      IF (.NOT.ACTIVE(5)) GOTO 5005

      G5 = GMAX - G
      CONSTR(5) = G5

 5005 CONTINUE

C     TO THE REFUELING POINT. THIS CONSTRAINT WILL NEVER BE BINDING
```

```
C     SO IT WILL BE REPLACED WITH A QUICK EQUATION THAT WILL GIVE THE
C     AMOUNT OF FUEL TRANSFERRED TO THE AIRLIFTER

      IF (.NOT.ACTIVE(6)) GOTO 5006

C     G6 = H - FRTA
C
      G6 = FCC + FRC - G

      CONSTR(6)=G6

 5006 CONTINUE

C     THE CARGO A/C MUST BE WITHIN RANGE OF THE ALTERNATE AT THE
C     REFUELING POINT

      IF (.NOT.ACTIVE(7)) GOTO 5007

      G7 = WFL - FRD
      CONSTR(7)=G7

 5007 CONTINUE


C     THE SUM OF H AND G IS ALWAYS GREATER THAN OR EQUAL TO THE
C     TOTAL FUEL REQUIRED FOR THE MISSION.

      IF (.NOT.ACTIVE(8)) GOTO 5008

      G8 = H + G - FRC - FCC - FRT - FCT
      CONSTR(8) = G8

 5008 CONTINUE

C     THIS CONSTRAINT IS SUPPOSED TO FORCE THE TANKER TO BRING
C     ENOUGH FUEL FOR THE TRANSPORT AS WELL.

      IF (.NOT.ACTIVE(9)) GOTO 5009

      G9 = GMAX + HMAX - FCCA - FCTA - FRC - FRT
      CONSTR(9)=G9

 5009 CONTINUE

      RETURN
      END
C
C   END OF NLFUNC
C
```

```
      SUBROUTINE NLGRAD(NOCONS,NOEQCO,NOMMAX,NOVARI,OBJFUN,
     1    CONSTR,GRADOF,GRADCO,VARIAB,ACTIVE,CONEPS)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CONSTR(NOMMAX),GRADOF(NOVARI),GRADCO(NOMMAX,NOVARI),
     1    VARIAB(NOVARI),CONEPS(NOMMAX)
      LOGICAL ACTIVE(NOMMAX)
C
C   EVALUATION OF GRADIENTS
C
      ON=1.D+0
      EPS=1.D-7
      DO1 I=1,NOVARI
      XEPS=EPS*DMAX1(ON,DABS(VARIAB(I)))
      XEPSI=ON/XEPS
      VARIAB(I)=VARIAB(I) + XEPS
      CALL NLFUNC(NOCONS,NOEQCO,NOMMAX,NOVARI,FEPS,CONEPS,VARIAB,
     1     ACTIVE)
      GRADOF(I)=(FEPS - OBJFUN)*XEPSI
      DO2 J=1,NOCONS
      IF (.NOT.ACTIVE(J)) GOTO 2
      GRADCO(J,I)=(CONEPS(J) - CONSTR(J))*XEPSI
    2 CONTINUE
    1 VARIAB(I)=VARIAB(I) - XEPS
C
C   END OF NLGRAD
C
      RETURN
      END
```

```
C*********************************************************************
C WHAT FOLLOWS IS THE RAW DATA OBTAINED BY RUNNING THE DIFFERENT
C VERSIONS OF THE MODEL. IF THE CODE IS BEING RECREATED, DO NOT CODE
C THE FOLLOWING MATERIAL:

 FCC =      62.247692095636
 FRC =      71.072002446404
 FCT =      3.5342247388147
 FRT =      5.2000000000001
 FCC + FRC - G =      13.372602881018


     * FINAL CONVERGENCE ANALYSIS

        OBJECTIVE FUNCTION VALUE:  F(X) =  0.14205392D+03
        APPROXIMATION OF SOLUTION:  X =
           0.11994709D+03  0.22106590D+02  0.26749665D+02  0.40339093D+02
        APPROXIMATION OF MULTIPLIERS:  U =
           0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
           0.00000000D+00  0.00000000D+00  0.10471249D+00  0.14171076D-01
           0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
           0.36898763D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
           0.00000000D+00
        CONSTRAINT VALUES:  G(X) =
           0.58167308D+02  0.49342998D+02  0.19126578D+03  0.62247689D+02
           0.46790834D+00  0.13372603D+02 -0.35763955D-05 -0.23727216D-03
           0.17427818D+03
        DISTANCE FROM LOWER BOUND:  XL-X =
          -0.11994709D+03 -0.22106590D+02 -0.39749665D+02 -0.53390926D+01
        DISTANCE FROM UPPER BOUND:  XU-X =
           0.88005291D+03  0.17789341D+03  0.48250335D+02  0.48660907D+02
        NUMBER OF FUNC-CALLS:  NFUNC = 101
        NUMBER OF GRAD-CALLS:  NGRAD =  45
        NUMBER OF QL-CALLS:    NQL   =  45
```

```
C   CLAYTON PFLIEGER/DR SCHITTKOWSKY DEC 1992
C   THIS IS MODEL4.F
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION VARIAB(5),CONSTR(10)
      DIMENSION GRADOF(5),GRADCO(10,5)
      DIMENSION HESSEM(5,5),RHSIDE(5)
      DIMENSION VECMUL(19),BOULOW(5),BOUUPP(5)
      DIMENSION WORKAR(527),IWORKA(39)
      LOGICAL ACTIVE(40)
      COMMON/CMACHE/EPS100,EPS200,EPS300
      OPEN(10,FILE='EMPAUXI.DAT')

C         1         2         3         4         5         6         7
C23456789012345678901234567890123456789012345678901234567890123456789012


      EPS100=1.D-13
      EPS200=1.D-7
      EPS300=1.D-3
      IOUTST=6
      ACCURA=1.D-7
      MAXITE=80
      MAXFUN=16
      SCABOU=1.D+3
      IPRINT=1
      INFAIL=0
      MODEAL=0
      NOVARI=4
      NOCONS=9
      NOEQCO=0
      NOMMAX=10
      NONMAX=5
      NOMNN2=19
      LEWORK=527
      LEIWOR=39
      LEACTI=40

      EWT = 100.0
      AXTKR = 300.0
      HMAX = AXTKR - EWT


C     THIS SETS THE UPPER BOUND, THE LOWER BOUND, AND THE
```

```
C     INITIAL GUESS OF THE DECISION VARIABLES.

      VARIAB(1) =   50.00
      BOULOW(1) =    0.0
      BOUUPP(1) = 1000.0

      VARIAB(2) =   50.00
      BOULOW(2) =    0.0
      BOUUPP(2) =   HMAX

      VARIAB(3) =   15.0
      BOULOW(3) =  -13.0
      BOUUPP(3) =   75.0

      VARIAB(4) =   45.0
      BOULOW(4) =   35.0
      BOUUPP(4) =   89.0

      CALL NLPQL1(NOCONS,NOEQCO,NOMMAX,NOVARI,NONMAX,NOMNN2,VARIAB,
     1  OBJFUN,CONSTR,GRADOF,GRADCO,VECMUL,BOULOW,BOUUPP,HESSEM,
     2  RHSIDE,ACCURA,SCABOU,MAXFUN,MAXITE,IPRINT,MODEAL,IOUTST,
     3  INFAIL,WORKAR,LEWORK,IWORKA,LEIWOR,ACTIVE,LEACTI,.FALSE.,
     4  .TRUE.)
C
C   OUTPUT ON RESULT
C
      WRITE(10,9020) INFAIL,IWORKA(1),IWORKA(2),IWORKA(4)
 9020 FORMAT(1X,4I10)
      DO 9000 I=1,NOVARI
      WRITE(10,9030) VARIAB(I)
      VALMUL=VECMUL(NOCONS+I)
      VALMU1=VECMUL(NOCONS+NOVARI+I)
      IF (VALMU1.GT.VALMUL) VALMUL=VALMU1
 9000 WRITE(10,9030) VALMUL
      WRITE(10,9030) OBJFUN
      SUMMUL=0.D+0
      MNNMUL=NOCONS + NOVARI + NOVARI
      DO 9001 J=1,MNNMUL
 9001 SUMMUL=SUMMUL + DABS(VECMUL(J))
      OBJFUN=0.D+0
      DO 9009 J=1,NOCONS
      GGGGGJ=DABS(CONSTR(J))
      IF (J.GT.NOEQCO.AND.CONSTR(J).GT.0.D+0) GGGGGJ=0.D+0
 9009 OBJFUN=OBJFUN + GGGGGJ
      DO 9002 J=1,9
      WRITE(10,9030) CONSTR(J)
 9002 WRITE(10,9030) VECMUL(J)
      WRITE(10,9030) OBJFUN
      WRITE(10,9030) SUMMUL
 9030 FORMAT(1X,D19.8)
      STOP
```

```
      END
C
C END OF MAIN PROGRAMM
C

      SUBROUTINE NLFUNC(NOCONS,NOEQCO,NOMMAX,NOVARI,OBJFUN,CONSTR,
     1     VARIAB,ACTIVE)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CONSTR(NOMMAX),VARIAB(NOVARI)
      LOGICAL ACTIVE(NOMMAX)

C     THESE ARE AIRCRAFT PERFORMANCE CONSTANTS.

      A0 = 45.9127
      A1 = -0.0531
      B0 = 74.9700
      B1 = -0.1353
      C1 = 0.015
      C0 = -0.08
      D1 = 0.016
      D0 = -0.127

      EWC   = 152.685
      EWT   = 100.0
      V     =   0.0
      W     =  50.0
      AXCGO = 323.1
      AXTKR = 300.0


C     GMAX AND HMAX ARE THE MAXIMUM INITIAL FUEL LOADS OF THE
C     CARGO A/C AND THE TANKER RESPECTIVELY.

      GMAX = AXCGO - EWC - W
      HMAX = AXTKR - EWT


C     R*MAX IS THE MAXIMUM RANGE OF THE * A/C

      RCMAX = (A0 + A1*(EWC + W + GMAX/2))*GMAX
      RTMAX = (B0 + B1*(EWT +  HMAX/2))*HMAX



      DESC1 = 1.2
      DESC2 = 1.2
      PAL1  = 1.3
      PAL2  = 1.0
      RES1  = 6.7
      RES?  = 3.0
```

```
      RFC   = 9.0
      RFT   = 0.75



      PI  = 3.141592654
      DTR = PI/180.0

      OLONG = -12.6  *DTR
      OLAT  =  46.03 *DTR

      DLONG =  74.6  *DTR
      DLAT  =  40.02 *DTR

      TLONG =  27.1  *DTR
      TLAT  =  38.7  *DTR

      ALONG = -12.6  *DTR
      ALAT  =  46.03 *DTR

      R = 3443.92
      ALT = 31000.0
      ALT   = ALT/6076.12

      RA = 60.0*((R + ALT)/R)

C     RD IS THE REFUELING DISTANCE

      RD = 200.0

      G   = VARIAB(1)
      H   = VARIAB(2)
      TH  = VARIAB(3)
      FI  = VARIAB(4)

C     HERE WE NEED TO FIND RPLAT AND RPLONG SUCH THAT THIS
C     IS A POINT 200 MILES ALONG THE GREAT CIRCLE ARC FROM
C     THE REFUELING POINT TO THE DESTINATION. NOTE: THIS CODE
C     IS FOR A WESTWARD FLIGHT PLAN ONLY. IT WOULD HAVE TO BE
C     MODIFIED TO WORK FOR AN EASTWARD FLIGHT PLAN.

C     WE START BY WORKING WITH THE BIG TRIANGLE.

C     A, B, C ARE THE NECESSARY SIDES OF THE BIG TRIANGLE

      A     = (90.0 - FI)*DTR

      B = DSIN(DLAT)*DSIN(FI*DTR)+DCOS(DLONG - TH*DTR)
     /     *DCOS(DLAT)*DCOS(DTR*FI)
      B = DACOS(B)
```

```
      C = (PI/2) - DLAT

C     BB AND CC ARE THE NECESSARY ANGLES OF THE BIG TRIANGLE

      BB   = DLONG - TH*DTR

      TEMP = (DCOS(C) - DCOS(A)*DCOS(B))/(DSIN(A)*DSIN(B))
      CC = DACOS(TEMP)

C     NOW WE MOVE TO THE SMALL TRIANGLE. HERE ARE THE SIDES, NOTE
C     THAT SIDE A IS THE SAME.

      BS = (RD/RA)*DTR
      CS = DACOS(DCOS(A)*DCOS(BS) + DSIN(A)*DSIN(BS)*DCOS(CC))

C     HERE IS THE ONE ANGLE OF THE SMALL TRIANGLE THAT WE NEED:

      TEMP = (DCOS(BS) - DCOS(CS)*DCOS(A))/(DSIN(CS)*DSIN(A))
      BBS = DACOS(TEMP)

C****************************************

C     IN THE CASE OF AN EAST -> WEST FLIGHT (DLONG > OLONG ), RPLONG
C     SHOULD BE:

      RPLONG = TH*DTR + BBS
      IF (DLONG .GT. OLONG) GOTO 55

C     HOWEVER, IF THE AIRLIFTER IS GOING WEST -> EAST, RPLONG SHOULD
C     BE FOUND BY LINE 54.

54    RPLONG = TH*DTR - BBS
55    CONTINUE

C****************************************



      RPLAT  = FI*DTR + (A - CS)


C     DOR IS THE GREAT CIRCLE DISTANCE FROM THE ORIGIN TO
C     THE REFUELING POINT. THE CONDITION THAT DOR BE AT LEAST 100
C     NAUTICAL MILES IS INCLUDED.

      DOR = DSIN(OLAT)*DSIN(FI*DTR)+DCOS(DTR*TH - OLONG)
     /     *DCOS(OLAT)*DCOS(DTR*FI)
      DOR = (R+ALT)*DACOS(DOR)
```

```
      IF (DOR .GT. 100.0) GOTO 100

      DOR = 100.0

  100 CONTINUE


C     DRD IS THE GREAT CIRCLE DISTANCE FROM THE REFUELING
C     POINT TO THE DESTINATION OF THE CARGO A/C

      DRD = DSIN(DLAT)*DSIN(RPLAT)+DCOS(DLONG - RPLONG)
     /     *DCOS(DLAT)*DCOS(RPLAT)
      DRD = (R+ALT)*DACOS(DRD)

      IF (DRD .GT. 100.0) GOTO 101

      DRD = 100.0

  101 CONTINUE


C     DBR IS THE GREAT CIRCLE DISTANCE FROM THE TANKER BASE
C     TO THE REFUELING POINT.

      DBR = DSIN(TLAT)*DSIN(FI*DTR)+DCOS(TH*DTR - TLONG)
     /     *DCOS(TLAT)*DCOS(DTR*FI)
      DBR = (R+ALT)*DACOS(DBR)

      IF (DBR .GT. 100.0) GOTO 102

      DBR = 100.0

  102 CONTINUE


C     DBR IS THE GREAT CIRCLE DISTANCE FROM THE REFUELING POINT TO
C     THE TANKER BASE.

      DRB = DSIN(TLAT)*DSIN(RPLAT)+DCOS(TLONG - RPLONG)
     /     *DCOS(TLAT)*DCOS(RPLAT)
      DRB = (R+ALT)*DACOS(DRB)

      IF (DRB .GT. 100.0) GOTO 103

      DRB = 100.0

  103 CONTINUE


C     DRA IS THE GREAT CIRCLE DISTANCE FROM THE REFUELING POINT TO
```

```
C     THE ALTERNATE.

      DRA = DSIN(ALAT)*DSIN(FI*DTR)+DCOS(ALONG - TH*DTR)
     /     *DCOS(ALAT)*DCOS(DTR*FI)
      DRA = (R+ALT)*DACOS(DRA)

      IF (DRA .GT. 100.0) GOTO 104

      DRA = 100.0

 104  CONTINUE


C     FCC IS THE FUEL CONSUMED BY THE CARGO AIRCRAFT IN REACHING
C     THE REFUELING POINT.

      FCC = G + (A0 + A1*(EWC + W))/A1
      FCC = FCC - (((A0+A1*(EWC+W+G))**2-2*A1*DOR)**0.5D0)/A1
      FCC = FCC + C1*(EWC + W + G) + C0


C     FCCA IS THE FUEL CONSUMED BY THE CARGO AIRCRAFT IF IT WERE
C     TO FLY FROM ITS ORIGIN TO THE REFUELING POINT WITH FULL
C     TANKS

      FCCA = GMAX + (A0 + A1*(EWC + W))/A1
      FCCA = FCCA - (((A0+A1*(EWC+W+GMAX))**2-2*A1*DOR)**0.5D0)/A1
      FCCA = FCCA + C1*(EWC + W + GMAX) + C0


C     FRC IS THE FUEL REQUIRED BY THE CARGO A/C TO FLY FROM THE
C     REFUELING POINT TO THE DESTINATION.


      DRDA = DRD - 100.0


      FRC=-EWC-W-A0/A1+(((A0+A1*(EWC+W))**2+2*A1*DRDA)**0.5D+0)/A1


      FRC = FRC + DESC1 + PAL1 + RES1 + RFC


C     FRD IS THE FUEL REQUIRED TO DIVERT TO THE ALTERNATE FROM
C     THE REFUELING POINT IF THE CARGO A/C IS NOT REFUELED

      DRAA = DRA - 100.0
      FRD=-EWC-W-A0/A1+(((A0+A1*(EWC+W))**2+2*A1*DRAA)**0.5D+0)/A1
```

```
      FRD = FRD + DESC1 + PAL1 + RES1


C     FRCA IS THE MINIMUM FUEL REQUIRED BY THE CARGO A/C TO FLY FROM
C     THE ORIGIN TO THE REFUELING POINT. FRCA .LE. FCC

      DORA = DOR
      FRCA=-EWC-W-A0/A1+(((A0+A1*(EWC+W))**2+2*A1*DORA)**0.5D+0)/A1
      FRCA = FRCA


C     FCT IS THE FUEL CONSUMED BY THE TANKER IN REACHING THE RE-
C     FUELING POINT.

      DBRA = DBR + 200
      FCT = H - (((B0+B1*(EWT+H))**2-2*B1*DBRA)**0.5D+0)/B1 +
     /     (B0 + B1*EWT)/B1
      FCT = FCT + D1*(EWT + H) + D0 + RFT


C     FCTA IS THE AMOUNT OF FUEL THE TANKER WOULD CONSUME IN FLYING
C     TO THE REFUELING POINT IF IT WERE TO TAKE OFF WITH FULL TANKS.

      FCTA=HMAX-(((B0+B1*(EWT+HMAX))**2-2*B1*DBRA)**0.5D+0)/B1
     /   + (B0 + B1*EWT)/B1
      FCTA = FCTA + D1*(EWT + HMAX) + D0 + RFT


C     FRT IS THE FUEL REQUIRED BY THE TANKER TO MAKE IT HOME FROM
C     THE REFUELING POINT.

      DRBA = DRB - 100.0
      FRT = -EWT - V - B0/B1 + (((B0+B1*(EWT+V))**2+2*B1*
     /     DRBA)**0.5D+0)/B1
      FRT = FRT + DESC2 + PAL2 + RES2


C     FRTA IS THE MINIMUM AMOUNT OF FUEL REQUIRED BY THE TANKER
C     TO FLY TWICE THE DISTANCE TO THE REFUELING POINT.

      FRTA = -EWT - V - B0/B1 + (((B0+B1*(EWT+V))**2+4*B1*
     /     DBR)**0.5D+0)/B1


C     WFL IS THE AMOUNT OF FUEL THAT THE CARGO A/C WILL HAVE LEFT
C     AT THE REFUELING POINT.

      WFL = G - FCC

      PRINT*,' -------------------------'
```

```
      PRINT*,' DOR      = ',DOR
      PRINT*,' DRA      = ',DRA
      PRINT*,' DBR      = ',DBR
      PRINT*,' DRB      = ',DRB
      PRINT*,' DRD      = ',DRD
      PRINT*,' RPLAT    = ',RPLAT/DTR
      PRINT*,' RPLONG   = ',RPLONG/DTR
      PRINT*,' FCC      = ',FCC
      PRINT*,' FRC      = ',FRC
      PRINT*,' FCT      = ',FCT
      PRINT*,' FRT      = ',FRT
      PRINT*,' TRANSFER = ',FCC+FRC-G
      PRINT*,' -------------------------'


C     V IS THE OBJECTIVE FUNCTION THAT WE SEEK TO MINIMIZE.
C     IT IS EQUAL TO THE SUM OF THE FUELS NECESSARY TO FLY
C     THE MISSION.

      V = FCC + FRC + FCT + FRT
      OBJFUN = V


C     THE FOLLOWING IS THE SET OF CONSTRAINTS FOR THIS NLP


C     DISTANCE FROM ORIGIN TO REFUEL POINT MUST BE LESS THAN
C     THE MAXIMUM RANGE OF THE CARGO AIRCRAFT.

      IF (.NOT.ACTIVE(1)) GOTO 5001

      G1 = GMAX - FCC
      CONSTR(1) = G1

 5001 CONTINUE


C     THE DISTANCE FROM THE REFUELING POINT TO THE DESTINATION
C     MUST BE LESS THAN THE MAXIMUM RANGE OF THE CARGO A/C

      IF (.NOT.ACTIVE(2)) GOTO 5002

      G2 = GMAX - (FRC)
      CONSTR(2) = G2

 5002 CONTINUE

C     THE DISTANCE FROM THE TANKER BASE TO THE REFUELING POINT
```

```
C     MUST BE LESS THAN THE TANKER'S OUT AND BACK RANGE.

      IF (.NOT.ACTIVE(3)) GOTO 5003

      G3 = HMAX - (FCT + FRT)
      CONSTR(3) = G3

 5003 CONTINUE


C     THE INITIAL FUEL G MUST BE AT LEAST ENOUGH TO GET THE
C     CARGO A/C TO THE REFUELING POINT.

      IF (.NOT.ACTIVE(4)) GOTO 5004

      G4 = G - FRCA
      CONSTR(4) = G4

 5004 CONTINUE


C     THE INITIAL FUEL MUST BE LESS THAN THE MAXIMUM AMOUNT THE
C     CARGO A/C CAN CARRY.

      IF (.NOT.ACTIVE(5)) GOTO 5005

      G5 = GMAX - G
      CONSTR(5) = G5

 5005 CONTINUE

C     TO THE REFUELING POINT. THIS CONSTRAINT WILL NEVER BE BINDING
C     SO IT WILL BE REPLACED WITH A QUICK EQUATION THAT WILL GIVE THE
C     AMOUNT OF FUEL TRANSFERRED TO THE AIRLIFTER

      IF (.NOT.ACTIVE(6)) GOTO 5006

C     G6 = H - FRTA
C
      G6 = FCC + FRC - G

      CONSTR(6)=G6

 5006 CONTINUE


C     THE CARGO A/C MUST BE WITHIN RANGE OF THE ALTERNATE AT THE
C     REFUELING POINT

      IF (.NOT.ACTIVE(7)) GOTO 5007
```

```
      G7 = WFL - FRD
      CONSTR(7)=G7

 5007 CONTINUE


C     THE SUM OF H AND G IS ALWAYS GREATER THAN OR EQUAL TO THE
C     TOTAL FUEL REQUIRED FOR THE MISSION.

      IF (.NOT.ACTIVE(8)) GOTO 5008

      G8 = H + G - FRC - FCC - FRT - FCT
      CONSTR(8) = G8

 5008 CONTINUE

C     THIS CONSTRAINT IS SUPPOSED TO FORCE THE TANKER TO BRING
C     ENOUGH FUEL FOR THE TRANSPORT AS WELL.

      IF (.NOT.ACTIVE(9)) GOTO 5009

      G9 = CMAX + HMAX - FCCA - FCTA - FRC - FRT
      CONSTR(9)=G9

 5009 CONTINUE

      RETURN
      END
C
C   END OF NLFUNC
C


      SUBROUTINE NLGRAD(NOCONS,NOEQCO,NOMMAX,NOVARI,OBJFUN,
     1    CONSTR,GRADOF,GRADCO,VARIAB,ACTIVE,CONEPS)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CONSTR(NOMMAX),GRADOF(NOVARI),GRADCO(NOMMAX,NOVARI),
     1    VARIAB(NOVARI),CONEPS(NOMMAX)
      LOGICAL ACTIVF(NOMMAX)
C
C   EVALUATION OF GRADIENTS
C
      ON=1.D+0
      EPS=1.D-7
      DO1 I=1,NOVARI
      XEPS=EPS*DMAX1(ON,DABS(VARIAB(I)))
      XEPSI=ON/XEPS
      VARIAB(I)=VARIAB(I) + XEPS
      CALL NLFUNC(NOCONS,NOEQCO,NOMMAX,NOVARI,FEPS,CONEPS,VARIAB,
```

```
     1     ACTIVE)
      GRADOF(I)=(FEPS - OBJFUN)*XEPSI
      DO2 J=1,NOCONS
      IF (.NOT.ACTIVE(J)) GOTO 2
      GRADCO(J,I)=(CONEPS(J) - CONSTR(J))*XEPSI
    2 CONTINUE
    1 VARIAB(I)=VARIAB(I) - XEPS
C
C   END OF NLGRAD
C
      RETURN
      END
```

```
C*****************************************************************
C WHAT FOLLOWS IS THE RAW DATA OBTAINED BY RUNNING THE DIFFERENT
C VERSIONS OF THE MODEL. IF THE CODE IS BEING RECREATED, DO NOT CODE
C THE FOLLOWING MATERIAL:

------------------------
DOR      =    1679.6930834380
DRA      =    1679.6930834380
DBR      =    125.45417940967
DRB      =    171.51457836147
DRD      =    2049.0568612162
RPLAT    =    40.970967346561
RPLONG   =    29.341343276227
FCC      =    59.810105482947
FRC      =    76.189514516955
FCT      =    8.4915441889549
FRT      =    6.3654698683336
TRANSFER =    20.409335928383
------------------------


  * FINAL CONVERGENCE ANALYSIS

     OBJECTIVE FUNCTION VALUE:  F(X) =  0.15085663D+03
     APPROXIMATION OF SOLUTION:  X =
        0.11559028D+03  0.35266349D+02  0.25110965D+02  0.40107726D+02
     APPROXIMATION OF MULTIPLIERS:  U =
        0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
        0.00000000D+00  0.00000000D+00  0.89031531D-01  0.30329188D-01
        0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
        0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
        0.00000000D+00
     CONSTRAINT VALUES:  G(X) =
        0.60604891D+02  0.44225483D+02  0.18514299D+03  0.65942178D+02
        0.48247159D+01  0.20409341D+02 -0.27981883D-09 -0.12759216D-08
        0.16281270D+03
     DISTANCE FROM LOWER BOUND:  XL-X =
       -0.11559028D+03 -0.35266349D+02 -0.38110965D+02 -0.51077260D+01
     DISTANCE FROM UPPER BOUND:  XU-X =
        0.88440972D+03  0.16473365D+03  0.49889035D+02  0.48892274D+02
     NUMBER OF FUNC-CALLS:  NFUNC =  26
     NUMBER OF GRAD-CALLS:  NGRAD =  25
     NUMBER OF QL-CALLS:    NQL   =  25
```

Appendix F. Model 5 and maxACL code

```
C   CLAYTON PFLIEGER/DR SCHITTKOWSKY JAN 1993
C   THIS IS MODEL5.F
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION VARIAB(5),CONSTR(10)
      DIMENSION GRADOF(5),GRADCO(10,5)
      DIMENSION HESSEM(5,5),RHSIDE(5)
      DIMENSION VECMUL(19),BOULOW(5),BOUUPP(5)
      DIMENSION WORKAR(527),IWORKA(39)
      LOGICAL ACTIVE(40)
      COMMON/CMACHE/EPS100,EPS200,EPS300
      OPEN(10,FILE='EMPAUXI.DAT')

C        1         2         3         4         5         6         7
C23456789012345678901234567890123456789012345678901234567890123456789012


      EPS100=1.D-13
      EPS200=1.D-7
      EPS300=1.D-3
      IOUTST=6
      ACCURA=1.D-7
      MAXITE=80
      MAXFUN=16
      SCABOU=1.D+3
      IPRINT=1
      INFAIL=0
      MODEAL=0
      NOVARI=4
      NOCONS=9
      NOEQCO=0
      NOMMAX=10
      NONMAX=5
      NOMNN2=19
      LEWORK=527
      LEIWOR=39
      LEACTI=40

      EWT = 100.0
      AXTKR = 300.0
      HMAX = AXTKR - EWT

C     THIS SETS THE UPPER BOUND, THE LOWER BOUND, AND THE
```

```
C     INITIAL GUESS OF THE DECISION VARIABLES.

      VARIAB(1) =   50.0
      BOULOW(1) =    0.0
      BOUUPP(1) =  200.0

      VARIAB(2) =   50.0
      BOULOW(2) =    0.0
      BOUUPP(2) =   HMAX

      VARIAB(3) =   15.0
      BOULOW(3) =  -13.0
      BOUUPP(3) =   75.0

      VARIAB(4) =   45.0
      BOULOW(4) =   35.0
      BOUUPP(4) =   89.0


      CALL NLPQL1(NOCONS,NOEQCO,NOMMAX,NOVARI,NONMAX,NOMNN2,VARIAB,
     1  OBJFUN,CONSTR,GRADOF,GRADCO,VECMUL,BOULOW,BOUUPP,HESSEM,
     2  RHSIDE,ACCURA,SCABOU,MAXFUN,MAXITE,IPRINT,MODEAL,IOUTST,
     3  INFAIL,WORKAR,LEWORK,IWORKA,LEIWOR,ACTIVE,LEACTI,.FALSE.,
     4  .TRUE.)
C
C   OUTPUT ON RESULT
C
      WRITE(10,9020) INFAIL,IWORKA(1),IWORKA(2),IWORKA(4)
 9020 FORMAT(1X,4I10)
      DO 9000 I=1,NOVARI
      WRITE(10,9030) VARIAB(I)
      VALMUL=VECMUL(NOCONS+I)
      VALMU1=VECMUL(NOCONS+NOVARI+I)
      IF (VALMU1.GT.VALMUL) VALMUL=VALMU1
 9000 WRITE(10,9030) VALMUL
      WRITE(10,9030) OBJFUN
      SUMMUL=0.D+0
      MNNMUL=NOCONS + NOVARI + NOVARI
      DO 9001 J=1,MNNMUL
 9001 SUMMUL=SUMMUL + DABS(VECMUL(J))
      OBJFUN=0.D+0
      DO 9009 J=1,NOCONS
      GGGGGJ=DABS(CONSTR(J))
      IF (J.GT.NOEQCO.AND.CONSTR(J).GT.0.D+0) GGGGGJ=0.D+0
 9009 OBJFUN=OBJFUN + GGGGGJ
      DO 9002 J=1,9
      WRITE(10,9030) CONSTR(J)
 9002 WRITE(10,9030) VECMUL(J)
      WRITE(10,9030) OBJFUN
      WRITE(10,9030) SUMMUL
 9030 FORMAT(1X,D19.8)
```

```
      STOP
      END
C
C END OF MAIN PROGRAMM
C


      SUBROUTINE NLFUNC(NOCONS,NOEQCO,NOMMAX,NOVARI,OBJFUN,CONSTR,
     1     VARIAB,ACTIVE)
      IMPLICIT DOUBLE PRECISION(A-H,L,O-Z)
      DIMENSION CONSTR(NOMMAX),VARIAB(NOVARI)
      LOGICAL ACTIVE(NOMMAX)

C     CONSTANTS**************************************************

C     THESE ARE AIRCRAFT PERFORMANCE CONSTANTS.

      G   = VARIAB(1)
      H   = VARIAB(2)
      TH  = VARIAB(3)
      FI  = VARIAB(4)

      A0 = 45.9127
      A1 = -0.0531
      B0 = 74.9700
      B1 = -0.1353
      C1 = 0.015
      C0 = -0.08
      D1 = 0.016
      D0 = -0.127

      EWC = 152.685
      EWT = 100.0
      V = 0.0
      AXCGO = 323.1
      AXTKR = 300.0
      CTAS = 435.0
      TTAS = 440.0

C*********************************************************************
C REMOVING THE COMMENT CHARACTERS FROM THIS DEFINITION OF W AND
C ADDING COMMENT CHARACTERS TO THE FOLLOWING DEFINITION IS NECESSARY
C TO CHANGE THIS FORTRAN CODE FROM MODEL 5 -- THE MODEL THAT
C FINDS THE MINIMUM FUEL FOR A FIXED CARGO WEIGHT, TO MAXACL --
C THE MODEL THAT FINDS THE MAXIMUM CARGO LOAD.
C
C     FOR MAXACL:
C
      W = AXCGO - EWC - G
C
C     FOR MODEL 5:
```

```
C
C     W = 25.0
C
C     REMEMBER, ONLY ONE OF THESE DEFINITIONS SHOULD BE ACTIVE. ALSO,
C     DON'T FORGET TO CHANGE THE OBJECTIVE FUNCTION LATER IN THE CODE.
C*********************************************************************


C     GMAX AND HMAX ARE THE MAXIMUM INITIAL FUEL LOADS OF THE
C     CARGO A/C AND THE TANKER RESPECTIVELY.

      GMAX = AXCGO - EWC - W
      HMAX = AXTKR - EWT


C     R*MAX IS THE MAXIMUM RANGE OF THE * A/C

      RCMAX = (A0 + A1*(EWC + W + GMAX/2))*GMAX
      RTMAX = (B0 + B1*(EWT +  HMAX/2))*HMAX

      DESC1 = 1.2
      DESC2 = 1.2
      PAL1  = 1.3
      PAL2  = 1.0
      RES1  = 6.7
      RES2  = 3.0

      RFC   = 9.0
      RFT   = 0.75

      PI  = 3.141592654
      DTR = PI/180.0


      OLONG = -12.6  *DTR
      OLAT  =  46.03 *DTR

      DLONG =  74.6  *DTR
      DLAT  =  40.02 *DTR

      TLONG =  27.1  *DTR
      TLAT  =  38.7  *DTR

      ALONG = -12.6  *DTR
      ALAT  =  46.03 *DTR

      R = 3443.92
      ALT = 31000.0
      ALT = ALT/6076.12
```

```
      HWIND = 83.0*DTR
      VWIND = 55.0

      RA = 60.0*((R + ALT)/R)

C     RD IS THE REFUELING DISTANCE

      RD = 200.0

C     END CONSTANTS*******************************  ************

C     HERE WE NEED TO FIND RPLAT AND RPLONG SUCH THAT THIS
C     IS A POINT 200 MILES ALONG THE GREAT CIRCLE ARC FROM
C     THE REFUELING POINT TO THE DESTINATION. NOTE: THIS CODE
C     IS FOR A WESTWARD FLIGHT PLAN ONLY. IT WOULD HAVE TO BE
C     MODIFIED TO WORK FOR AN EASTWARD FLIGHT PLAN.

C     WE START BY WORKING WITH THE BIG TRIANGLE.

C     A, B, C ARE THE NECESSARY SIDES OF THE BIG TRIANGLE

      A    = (90.0 - FI)*DTR

      B = DSIN(DLAT)*DSIN(FI*DTR)+DCOS(DLONG - TH*DTR)
     /     *DCOS(DLAT)*DCOS(DTR*FI)
      B = DACOS(B)

      C = (PI/2) - DLAT

C     BB AND CC ARE THE NECESSARY ANGLES OF THE BIG TRIANGLE

      BB    = DLONG - TH*DTR

      TEMP = (DCOS(C) - DCOS(A)*DCOS(B))/(DSIN(A)*DSIN(B))
      CC = DACOS(TEMP)

C     NOW WE MOVE TO THE SMALL TRIANGLE. HERE ARE THE SIDES, NOTE
C     THAT SIDE A IS THE SAME.

      BS = (RD/RA)*DTR
      CS = DACOS(DCOS(A)*DCOS(BS) + DSIN(A)*DSIN(BS)*DCOS(CC))

C     HERE IS THE ONE ANGLE OF THE SMALL TRIANGLE THAT WE NEED:

      TEMP = (DCOS(BS) - DCOS(CS)*DCOS(A))/(DSIN(CS)*DSIN(A))
      BBS = DACOS(TEMP)
```

```
C****************************************

C     IN THE CASE OF AN EAST -> WEST FLIGHT (DLONG > OLONG ), RPLONG
C     SHOULD BE:

      RPLONG = TH*DTR + BBS
      IF (DLONG .GT. OLONG) GOTO 551

C     HOWEVER, IF THE AIRLIFTER IS GOING WEST -> EAST, RPLONG SHOULD
C     BE FOUND BY LINE 541.

541   RPLONG = TH*DTR - BBS
551   CONTINUE

C****************************************




      RPLAT  = FI*DTR + (A - CS)

C     THE NEXT THING TO DO IS TO FIND THE AVERAGE HEADING OVER
C     EACH LEG AND THEN SOLVE THE APPROPRIATE FUEL FUNCTIONS.


C     LEG 1  *****************************************************

C     DOR IS THE GREAT CIRCLE DISTANCE FROM THE ORIGIN TO
C     THE REFUELING POINT. THE CONDITION THAT DOR BE AT LEAST 100
C     NAUTICAL MILES IS INCLUDED.

      DOR = DSIN(OLAT)*DSIN(FI*DTR)+DCOS(DTR*TH - OLONG)
     /     *DCOS(OLAT)*DCOS(DTR*FI)
      ANGLE = DACOS(DOR)
      DOR = (R+ALT)*DACOS(DOR)

      IF (DOR .GT. 100.0) GOTO 100

      DOR = 100.0

 100  CONTINUE

      L1 = OLAT
      L2 = FI*DTR
      LONG1 = OLONG
      LONG2 = TH*DTR


C     THE HEADING EQUATION
```

```
      HDG = DSIN(L2) - DSIN(L1)*DCOS(ANGLE)
      HDG = HDG/(DSIN(ANGLE)*DCOS(L1))
      HDG = DACOS(HDG)

      IF ( DSIN(LONG2 - LONG1) .LT. 0.0 ) GOTO 50
      HDG = 2*PI - HDG
 50   CONTINUE


C     HI IS THE INITIAL HEADING OUT OF THE ORIGIN

      HI = HDG

C     NOW WE FIND THE FINAL HEADING.

      L1 = FI*DTR
      L2 = OLAT
      LONG1 = TH*DTR
      LONG2 = OLONG

      HDG = DSIN(L2) - DSIN(L1)*DCOS(ANGLE)
      HDG = HDG/(DSIN(ANGLE)*DCOS(L1))
      HDG = DACOS(HDG)

      IF ( DSIN(LONG2 - LONG1) .LT. 0.0 ) GOTO 51
      HDG = 2*PI - HDG
 51   CONTINUE


C     HF IS THE FINAL HEADING INTO THE ARCP

      HF = HDG + PI
      IF ( HF .LE. 2*PI ) GOTO 30
      HF = HF - 2*PI
 30   CONTINUE

C     THE AVERAGE HEADING

      HAVG = (HF + HI)/2



C     HERE WE ARE SOLVING A SINGLE OBLIQUE TRIANGLE IN ORDER TO
C     FIND THE GROUNDSPEED OF THE AIRCRAFT. AH, BH, & CH ARE THE
C     ANGLES OF THE TRIANGLE FORMED BY THE COURSE, THE WIND CORRECTED
C     HEADING (WITH MAGNITUDE OF CTAS OR TTAS AS APPROPRIATE) AND THE
C     WIND.

      BH = PI - DABS(PI - DABS(HAVG - HWIND))
      CH = DASIN(DSIN(BH)*(VWIND/CTAS))
```

```
      AH = PI - BH - CH
      GS = DSQRT(CTAS**2 + VWIND**2 - 2*CTAS*VWIND*DCOS(AH))
C     GS IS THE MAGNITUDE OF THE AIRCRAFT GROUND SPEED

C     THE FUEL MILEAGE CORRECTION FACTOR
      F = GS/CTAS



C     FCC IS THE FUEL CONSUMED BY THE CARGO AIRCRAFT IN REACHING
C     THE REFUELING POINT.

      FCC = G + (F*A0 + A1*(EWC + W))/A1
      FCC = FCC - (((F*A0+A1*(EWC+W+G))**2-2*A1*DOR)**0.5D0)/A1
      FCC = FCC + C1*(EWC + W + G) + C0


C     FCCA IS THE FUEL CONSUMED BY THE CARGO AIRCRAFT IF IT WERE
C     TO FLY FROM ITS ORIGIN TO THE REFUELING POINT WITH FULL
C     TANKS

      FCCA = GMAX + (F*A0 + A1*(EWC + W))/A1
      FCCA =FCCA-(((F*A0+A1*(EWC+W+GMAX))**2-2*A1*DOR)**0.5D0)/A1
      FCCA = FCCA + C1*(EWC + W + GMAX) + C0


C     FRCA IS THE MINIMUM FUEL REQUIRED BY THE CARGO A/C TO FLY FROM
C     THE ORIGIN TO THE REFUELING POINT. FRCA .LE. FCC

      DORA = DOR - 100.0
      FRCA=-EWC-W-F*A0/A1+(((F*A0+A1*(EWC+W))**2
     / +2*A1*DORA)**0.5D+0)/A1
      FRCA = FRCA + DESC1 + PAL1 + RES1


C     LEG 1A **************************************************

C     DRA IS THE GREAT CIRCLF DISTANCE FROM THE REFUELING POINT TO
C     THE ALTERNATE.

      DRA = DSIN(ALAT)*DSIN(FI*DTR)+DCOS(ALONG - TH*DTR)
     /      *DCOS(ALAT)*DCOS(DTR*FI)
      ANGLE = DACOS(DRA)
      DRA = (R+ALT)*DACOS(DRA)

      IF (DRA .GT. 100.0) GOTO 104

      DRA = 100.0

  104 CONTINUE
```

```
      L1 = FI*DTR
      L2 = OLAT
      LONG1 = TH*DTR
      LONG2 = OLONG

C     THE HEADING EQUATION

      HDG = DSIN(L2) - DSIN(L1)*DCOS(ANGLE)
      HDG = HDG/(DSIN(ANGLE)*DCOS(L1))
      HDG = DACOS(HDG)

      IF ( DSIN(LONG2 - LONG1) .LT. 0.0 ) GOTO 52
      HDG = 2*PI - HDG
 52   CONTINUE


C     HI IS THE INITIAL HEADING OUT OF THE ARCP

      HI = HDG

C     NOW WE FIND THE FINAL HEADING.

      L1 = OLAT
      L2 = FI*DTR
      LONG1 = OLONG
      LONG2 = TH*DTR

      HDG = DSIN(L2) - DSIN(L1)*DCOS(ANGLE)
      HDG = HDG/(DSIN(ANGLE)*DCOS(L1))
      HDG = DACOS(HDG)

      IF ( DSIN(LONG2 - LONG1) .LT. 0.0 ) GOTO 53
      HDG = 2*PI - HDG
 53   CONTINUE


C     HF IS THE FINAL HEADING INTO THE ALTERNATE


      HF = HDG + PI
      IF ( HF .LE. 2*PI ) GOTO 31
      HF = HF - 2*PI
 31   CONTINUE


C     THE AVERAGE HEADING

      HAVG = (HF + HI)/2
```

```
C     HERE WE ARE SOLVING A SINGLE OBLIQUE TRIANGLE IN ORDER TO
C     FIND THE GROUNDSPEED OF THE AIRCRAFT. AH, BH, & CH ARE THE
C     ANGLES OF THE TRIANGLE FORMED BY THE COURSE, THE WIND CORRECTED
C     HEADING (WITH MAGNITUDE OF CTAS OR TTAS AS APPROPRIATE) AND THE
C     WIND.

      BH = PI - DABS(PI - DABS(HAVG - HWIND))
      CH = DASIN(DSIN(BH)*(VWIND/CTAS))
      AH = PI - BH - CH
      GS = DSQRT(CTAS**2 + VWIND**2 - 2*CTAS*VWIND*DCOS(AH))
C     GS IS THE MAGNITUDE OF THE AIRCRAFT GROUND SPEED

C     THE FUEL MILEAGE CORRECTION FACTOR
      F = GS/CTAS




C     FRD IS THE FUEL REQUIRED TO DIVERT TO THE ALTERNATE FROM
C     THE REFUELING POINT IF THE CARGO A/C IS NOT REFUELED

      DRAA = DRA - 100.0
      FRD=-EWC-W-F*A0/A1+(((F*A0+A1*(EWC+W))**2
     /   +2*A1*DRAA)**0.5D+0)/A1
      FRD = FRD + DESC1 + PAL1 + RES1



C     LEG 2***************************************************

C     DBR IS THE GREAT CIRCLE DISTANCE FROM THE TANKER BASE
C     TO THE REFUELING POINT.

      DBR = DSIN(TLAT)*DSIN(FI*DTR)+DCOS(TH*DTR - TLONG)
     /     *DCOS(TLAT)*DCOS(DTR*FI)
      ANGLE = DACOS(DBR)
      DBR = (R+ALT)*DACOS(DBR)

      IF (DBR .GT. 100.0) GOTO 102

      DBR = 100.0

 102  CONTINUE

      L1 = TLAT
      L2 = FI*DTR
      LONG1 = TLONG
      LONG2 = TH*DTR
```

```
C     THE HEADING EQUATION

      HDG = DSIN(L2) - DSIN(L1)*DCOS(ANGLE)
      HDG = HDC/(DSIN(ANGLE)*DCOS(L1))
      HDG = DACOS(HDG)

      IF ( DSIN(LONG2 - LONG1) .LT. 0.0 ) GOTO 54
      HDG = 2*PI - HDG
 54   CONTINUE

C     HI IS THE INITIAL HEADING OUT OF THE TANKER BASE

      HI = HDG

C     NOW WE FIND THE FINAL HEADING.

      L1 = FI*DTR
      L2 = TLAT
      LONG1 = TH*DTR
      LONG2 = TLONG

      HDG = DSIN(L2) - DSIN(L1)*DCOS(ANGLE)
      HDG = HDG/(DSIN(ANGLE)*DCOS(L1))
      HDG = DACOS(HDG)

      IF ( DSIN(LONG2 - LONG1) .LT. 0.0 ) GOTO 55
      HDG = 2*PI - HDG
 55   CONTINUE

C     HF IS THE FINAL HEADING INTO THE ARCP


      HF = HDG + PI
      IF ( HF .LE. 2*PI ) GOTO 32
      HF = HF - 2*PI
 32   CONTINUE

C     THE AVERAGE HEADING

      HAVG = (HF + HI)/2

C     HERE WE ARE SOLVING A SINGLE OBLIQUE TRIANGLE IN ORDER TO
C     FIND THE GROUNDSPEED OF THE AIRCRAFT. AH, BH, & CH ARE THE
C     ANGLES OF THE TRIANGLE FORMED BY THE COURSE, THE WIND CORRECTED
C     HEADING (WITH MAGNITUDE OF CTAS OR TTAS AS APPROPRIATE) AND THE
C     WIND.
```

```
      BH = PI - DABS(PI - DABS(HAVG - HWIND))
      CH = DASIN(DSIN(BH)*(VWIND/TTAS))
      AH = PI - BH - CH
      GS = DSQRT(TTAS**2 + VWIND**2 - 2*TTAS*VWIND*DCOS(AH))
C     GS IS THE MAGNITUDE OF THE AIRCRAFT GROUND SPEED

C     THE FUEL MILEAGE CORRECTION FACTOR
      F = GS/TTAS




C     FCT IS THE FUEL CONSUMED BY THE TANKER IN REACHING THE RE-
C     FUELING POINT.

      DBRA = DBR + 200
      FCT = H - (((F*B0+B1*(EWT+H))**2-2*B1*DBRA)**0.5D+0)/B1
     /+(F*B0 + B1*EWT)/B1
      FCT = FCT + D1*(EWT + H) + D0 + RFT


C     FCTA IS THE AMOUNT OF FUEL THE TANKER WOULD CONSUME IN FLYING
C     TO THE REFUELING POINT IF IT WERE TO TAKE OFF WITH FULL TANKS.

      FCTA=HMAX-(((F*B0+B1*(EWT+HMAX))**2-2*B1*DBRA)**0.5D+0)/B1
     /+(F*B0 + B1*EWT)/B1
      FCTA = FCTA + D1*(EWT + HMAX) + D0 + RFT



C     LEG 3**************************************************

C     DRB IS THE GREAT CIRCLE DISTANCE FROM THE REFUELING POINT TO
C     THE TANKER BASE.

      DRB = DSIN(TLAT)*DSIN(RPLAT)+DCOS(TLONG - RPLONG)
     /     *DCOS(TLAT)*DCOS(RPLAT)
      ANGLE = DACOS(DRB)
      DRB = (R+ALT)*DACOS(DRB)

      IF (DRB .GT. 100.0) GOTO 103

      DRB = 100.0

 103  CONTINUE

      L1 = RPLAT
      L2 = TLAT
      LONG1 = RPLONG
      LONG2 = TLONG
```

```
C     THE HEADING EQUATION

      HDG = DSIN(L2) - DSIN(L1)*DCOS(ANGLE)
      HDG = HDG/(DSIN(ANGLE)*DCOS(L1))
      HDG = DACOS(HDG)

      IF ( DSIN(LONG2 - LONG1) .LT. 0.0 ) GOTO 56
      HDG = 2*PI - HDG
 56   CONTINUE


C     HI IS THE INITIAL HEADING OUT OF THE ARCP

      HI = HDG

C     NOW WE FIND THE FINAL HEADING.

      L1 = TLAT
      L2 = RPLAT
      LONG1 = TLONG
      LONG2 = RPLONG

      HDG = DSIN(L2) - DSIN(L1)*DCOS(ANGLE)
      HDG = HDG/(DSIN(ANGLE)*DCOS(L1))
      HDG = DACOS(HDG)

      IF ( DSIN(LONG2 - LONG1) .LT. 0.0 ) GOTO 57
      HDG = 2*PI - HDG
 57   CONTINUE


C     HF IS THE FINAL HEADING INTO THE TANKER BASE


      HF = HDG + PI
      IF ( HF .LE. 2*PI ) GOTO 33
      HF = HF - 2*PI
 33   CONTINUE


C     THE AVERAGE HEADING

      HAVG = (HF + HI)/2

C     HERE WE ARE SOLVING A SINGLE OBLIQUE TRIANGLE IN ORDER TO
C     FIND THE GROUNDSPEED OF THE AIRCRAFT. AH, BH, & CH ARE THE
C     ANGLES OF THE TRIANGLE FORMED BY THE COURSE, THE WIND CORRECTED
C     HEADING (WITH MAGNITUDE OF CTAS OR TTAS AS APPROPRIATE) AND THE
C     WIND.
```

```
      BH = PI - DABS(PI - DABS(HAVG - HWIND))
      CH = DASIN(DSIN(BH)*(VWIND/TTAS))
      AH = PI - BH - CH
      GS = DSQRT(TTAS**2 + VWIND**2 - 2*TTAS*VWIND*DCOS(AH))
C     GS IS THE MAGNITUDE OF THE AIRCRAFT GROUND SPEED

C     THE FUEL MILEAGE CORRECTION FACTOR
      F = GS/TTAS




C     FRT IS THE FUEL REQUIRED BY THE TANKER TO MAKE IT HOME FROM
C     THE REFUELING POINT.

      DRBA - DRB - 100.0
      FRT = -EWT -V-F*B0/B1+(((F*B0+B1*(EWT+V))**2+2*B1*
     /    DRBA)**0.5D+0)/B1
      FRT = FRT + DESC2 + PAL2 + RES2


C     LEG 4***************************************************


C     DRD IS THE GREAT CIRCLE DISTANCE FROM THE REFUELING
C     POINT TO THE DESTINATION OF THE CARGO A/C

      DRD = DSIN(DLAT)*DSIN(RPLAT)+DCOS(DLONG - RPLONG)
     /     *DCOS(DLAT)*DCOS(RPLAT)
      ANGLE = DACOS(DRD)
      DRD = (R+ALT)*DACOS(DRD)

      IF (DRD .GT. 100.0) GOTO 101

      DRD = 100.0

 101  CONTINUE


      L1 = RPLAT
      L2 = DLAT
      LONG1 = RPLONG
      LONG2 = DLONG

C     THE HEADING EQUATION

      HDG = DSIN(L2) - DSIN(L1)*DCOS(ANGLE)
      HDG = HDG/(DSIN(ANGLE)*DCOS(L1))
      HDG = DACOS(HDG)

      IF ( DSIN(LONG2 - LONG1) .LT. 0.0 ) GOTO 58
```

```
      HDG = 2*PI - HDG
 58   CONTINUE


C     HI IS THE INITIAL HEADING OUT OF THE EXIT POINT

      HI = HDG

C     NOW WE FIND THE FINAL HEADING.

      L1 = DLAT
      L2 = RPLAT
      LONG1 = DLONG
      LONG2 = RPLONG

      HDG = DSIN(L2) - DSIN(L1)*DCOS(ANGLE)
      HDG = HDG/(DSIN(ANGLE)*DCOS(L1))
      HDG = DACOS(HDG)

      IF ( DSIN(LONG2 - LONG1) .LT. 0.0 ) GOTO 59
      HDG = 2*PI - HDG
 59   CONTINUE


C     HF IS THE FINAL HEADING INTO THE DESTINATION


      HF = HDG + PI
      IF ( HF .LE. 2*PI ) GOTO 34
      HF = HF - 2*PI
 34   CONTINUE


C     THE AVERAGE HEADING

      HAVG = (HF + HI)/2

C     HERE WE ARE SOLVING A SINGLE OBLIQUE TRIANGLE IN ORDER TO
C     FIND THE GROUNDSPEED OF THE AIRCRAFT. AH, BH, & CH ARE THE
C     ANGLES OF THE TRIANGLE FORMED BY THE COURSE, THE WIND CORRECTED
C     HEADING (WITH MAGNITUDE OF CTAS OR TTAS AS APPROPRIATE) AND THE
C     WIND.

      BH = PI - DABS(PI - DABS(HAVG - HWIND))
      CH = DASIN(DSIN(BH)*(VWIND/CTAS))
      AH = PI - BH - CH
      GS = DSQRT(CTAS**2 + VWIND**2 - 2*CTAS*VWIND*DCOS(AH))
C     GS IS THE MAGNITUDE OF THE AIRCRAFT GROUND SPEED

C     THE FUEL MILEAGE CORRECTION FACTOR
      F = GS/CTAS
```

```
C     FRC IS THE FUEL REQUIRED BY THE CARGO A/C TO FLY FROM THE
C     REFUELING POINT TO THE DESTINATION.

      DRDA = DRD - 100.0
      FRC=-EWC-W-F*A0/A1+(((F*A0+A1*(EWC+W))**2
     / +2*A1*DRDA)**0.5D+0)/A1
      FRC = FRC + DESC1 + PAL1 + RES1 + RFC


C     LAST LEG!****************************************************


C     FRTA IS THE MINIMUM AMOUNT OF FUEL REQUIRED BY THE TANKER
C     TO FLY TWICE THE DISTANCE TO THE REFUELING POINT.

      FRTA = -EWT - V - B0/B1 + (((B0+B1*(EWT+V))**2+4*B1*
     /    DBR)**0.5D+0)/B1


C     WFL IS THE AMOUNT OF FUEL THAT THE CARGO A/C WILL HAVE LEFT
C     AT THE REFUELING POINT.

      WFL = G - FCC


      PRINT*,' ------------------------'
      PRINT*,' DOR      = ',DOR
      PRINT*,' DRA      = ',DRA
      PRINT*,' DBR      = ',DBR
      PRINT*,' DRB      = ',DRB
      PRINT*,' DRD      = ',DRD
      PRINT*,' RPLAT    = ',RPLAT/DTR
      PRINT*,' RPLONG   = ',RPLONG/DTR
      PRINT*,' FCC      = ',FCC
      PRINT*,' FRC      = ',FRC
      PRINT*,' FCT      = ',FCT
      PRINT*,' FRT      = ',FRT
      PRINT*,' TRANSFER = ',FCC+FRC-G
      PRINT*,' ------------------------'



C     ****************************************************
C     V IS THE OBJECTIVE FUNCTION THAT WE SEEK TO MINIMIZE.
C     IT IS EITHER EQUAL TO THE SUM OF THE FUELS NECESSARY
C     TO FLY THE MISSION OR IT IS  EQUAL TO the negative
C     of the maximum Allowable Cabin Weight.
C     FOR MODEL 5:
```

```
C
C      V = FCC + FRC + FCT + FRT
C
C     FOR MAXACL:
C
      V = G + EWC - AXCGO
C

      OBJFUN = V

C     ****************************************************


C     ****************************************************

C     THE FOLLOWING IS THE SET OF CONSTRAINTS FOR THIS NLP

C     ****************************************************


C     DISTANCE FROM ORIGIN TO REFUEL POINT MUST BE LESS THAN
C     THE MAXIMUM RANGE OF THE CARGO AIRCRAFT.

      IF (.NOT.ACTIVE(1)) GOTO 5001

      G1 = GMAX - FCC
      CONSTR(1) = G1

 5001 CONTINUE


C     THE DISTANCE FROM THE REFUELING POINT TO THE DESTINATION
C     MUST BE LESS THAN THE MAXIMUM RANGE OF THE CARGO A/C

      IF (.NOT.ACTIVE(2)) GOTO 5002

      G2 = GMAX - (FRC)
      CONSTR(2) = G2

 5002 CONTINUE

C     THE DISTANCE FROM THE TANKER BASE TO THE REFUELING POINT
C     MUST BE LESS THAN THE TANKER'S OUT AND BACK RANGE.

      IF (.NOT.ACTIVE(3)) GOTO 5003

      G3 = HMAX - (FCT + FRT)
      CONSTR(3) = G3
```

```
 5003 CONTINUE

C     THE INITIAL FUEL G MUST BE AT LEAST ENOUGH TO GET THE
C     CARGO A/C TO THE REFUELING POINT.

      IF (.NOT.ACTIVE(4)) GOTO 5004

      G4 = G - FRCA
      CONSTR(4) = G4

 5004 CONTINUE



C     THE INITIAL FUEL MUST BE LESS THAN THE MAXIMUM AMOUNT THE
C     CARGO A/C CAN CARRY.

      IF (.NOT.ACTIVE(5)) GOTO 5005

      G5 = GMAX - G
      CONSTR(5) = G5

 5005 CONTINUE

C     TO THE REFUELING POINT. THIS CONSTRAINT WILL NEVER BE BINDING
C     SO IT WILL BE REPLACED WITH A QUICK EQUATION THAT WILL GIVE THE
C     AMOUNT OF FUEL TRANSFERRED TO THE AIRLIFTER

      IF (.NOT.ACTIVE(6)) GOTO 5006

C     G6 = H - FRTA
C
      G6 = FCC + FRC - G

      CONSTR(6)=G6

 5006 CONTINUE


C     THE CARGO A/C MUST BE WITHIN RANGE OF THE ALTERNATE AT THE
C     REFUELING POINT

      IF (.NOT.ACTIVE(7)) GOTO 5007

      G7 = WFL - FRD
      CONSTR(7)=G7

 5007 CONTINUE
```

```
C     THE SUM OF H AND G IS ALWAYS GREATER THAN OR EQUAL TO THE
C     TOTAL FUEL REQUIRED FOR THE MISSION.

      IF (.NOT.ACTIVE(8)) GOTO 5008

      G8 = H + G - FRC - FCC - FRT - FCT
      CONSTR(8) = G8

 5008 CONTINUE

C     THIS CONSTRAINT IS SUPPOSED TO FORCE THE TANKER TO BRING
C     ENOUGH FUEL FOR THE TRANSPORT AS WELL.

      IF (.NOT.ACTIVE(9)) GOTO 5009

      G9 = GMAX + HMAX - FCCA - FCTA - FRC - FRT
      CONSTR(9)=G9

 5009 CONTINUE

      RETURN
      END
C
C   END OF NLFUNC
C


      SUBROUTINE NLGRAD(NOCONS,NOEQCO,NOMMAX,NOVARI,OBJFUN,
     1    CONSTR,GRADOF,GRADCO,VARIAB,ACTIVE,CONEPS)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CONSTR(NOMMAX),GRADOF(NOVARI),GRADCO(NOMMAX,NOVARI),
     1    VARIAB(NOVARI),CONEPS(NOMMAX)
      LOGICAL ACTIVE(NOMMAX)
C
C   EVALUATION OF GRADIENTS
C
      ON=1.D+0
      EPS=1.D-7
      DO1 I=1,NOVARI
      XEPS=EPS*DMAX1(ON,DABS(VARIAB(I)))
      XEPSI=ON/XEPS
      VARIAB(I)=VARIAB(I) + XEPS
      CALL NLFUNC(NOCONS,NOEQCO,NOMMAX,NOVARI,FEPS,CONEPS,VARIAB,
     1     ACTIVE)
      GRADOF(I)=(FEPS - OBJFUN)*XEPSI
      DO2 J=1,NOCONS
      IF (.NOT.ACTIVE(J)) GOTO 2
      GRADCO(J,I)=(CONEPS(J) - CONSTR(J))*XEPSI
    2 CONTINUE
    1 VARIAB(I)=VARIAB(I) - XEPS
```

```
C
C   END OF NLGRAD
C
      RETURN
      END
```

```
C*********************************************************************
C WHAT FOLLOWS IS THE RAW DATA OBTAINED BY RUNNING THE DIFFERENT
C VERSIONS OF THE MODEL. IF THE CODE IS BEING RECREATED, DO NOT CODE
C THE FOLLOWING MATERIAL:


RESULTS FOR MODEL 5:

 -----------------------
 DOR      =     1646.6925341404
 DRA      =     1646.6925341404
 DBR      =     159.83356934315
 DRB      =     176.64126112585
 DRD      =     2067.7449183072
 RPLAT    =     41.328658885010
 RPLONG   =     28.803310384926
 FCC      =     66.855688402272
 FRC      =     86.328757600773
 FCT      =     8.9186700024949
 FRT      =     6.3984856475219
 TRANSFER =     39.651520328763
 -----------------------


    * FINAL CONVERGENCE ANALYSIS

       OBJECTIVE FUNCTION VALUE:  F(X) =  0.16850160D+03
       APPROXIMATION OF SOLUTION:  X =
          0.11353293D+03  0.54968675D+02  0.24544149D+02  0.40480991D+02
       APPROXIMATION OF MULTIPLIERS:  U =
          0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
          0.00000000D+00  0.00000000D+00  0.12490441D+00  0.29483511D-01
          0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
          0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
          0.00000000D+00
       CONSTRAINT VALUES:  G(X) =
          0.78559308D+02  0.59086240D+02  0.18468285D+03  0.51515622D+02
          0.31882074D+02  0.39651526D+02 -0.44508397D-10 -0.29190339D-09
          0.16743605D+03
       DISTANCE FROM LOWER BOUND:  XL-X =
         -0.11353293D+03 -0.54968675D+02 -0.37544149D+02 -0.54809915D+01
       DISTANCE FROM UPPER BOUND:  XU-X =
          0.86467074D+02  0.14503132D+03  0.50455851D+02  0.48519009D+02
       NUMBER OF FUNC-CALLS:  NFUNC =  26
       NUMBER OF GRAD-CALLS:  NGRAD =  25
       NUMBER OF QL-CALLS:    NQL   =  25
```

RESULTS FOR MAXACL:

```
-----------------------
DOR       =     1292.8105807426
DRA       =     1292.8105807426
DBR       =     941.99401290880
DRB       =     890.32713348030
DRD       =     2164.9446501219
RPLAT     =     53.385543632806
RPLONG    =     24.538677954544
FCC       =     56.779802810952
FRC       =     97.183223506063
FCT       =     25.302559849492
FRT       =     18.896041005836
TRANSFER =      56.779801504775
-----------------------
```

```
* FINAL CONVERGENCE ANALYSIS

   OBJECTIVE FUNCTION VALUE:  F(X) = -0.73231775D+02
   APPROXIMATION OF SOLUTION:  X =
      0.97183225D+02  0.10097840D+03  0.18962234D+02  0.53318286D+02
   APPROXIMATION OF MULTIPLIERS:  U =
      0.00000000D+00  0.54002128D+00  0.00000000D+00  0.00000000D+00
      0.00000000D+00  0.00000000D+00  0.35027596D+00  0.00000000D+00
      0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
      0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
      0.00000000D+00
   CONSTRAINT VALUES:  G(X) =
      0.40403422D+02 -0.23163693D-11  0.15580141D+03  0.44580711D+02
      0.00000000D+00  0.56779802D+02 -0.11233681D-10  0.44026629D-05
      0.90545504D+02
   DISTANCE FROM LOWER BOUND:  XL-X =
     -0.97183225D+02 -0.10097840D+03 -0.31962234D+02 -0.18318286D+02
   DISTANCE FROM UPPER BOUND:  XU-X =
      0.10281678D+03  0.99021603D+02  0.56037766D+02  0.35681714D+02
   NUMBER OF FUNC-CALLS:  NFUNC =  13
   NUMBER OF GRAD-CALLS:  NGRAD =  13
   NUMBER OF QL-CALLS:    NQL   =  13
```

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Vita

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13. ABSTRACT *(Maximum 200 words)*

This research follows A. Yamani's work on the problem of minimizing total fuel costs for airlift missions that require in-flight refueling. Yamani's major assumptions were removed and the resulting enhanced formulation was applied the the original problem and the related problem of maximizing allowable cabin load. This optimization is accomplished selecting the rendezvous point and initial fuel for each aircraft. The formulation considers a C-141B airlifter refueled once by a KC-135E tanker. Flight is assumed to be at constant altitude and follow great-circle routes. The fuel costs of climb and cruise flight are modeled as functions of gross weight, while fuel costs associated with the air refueling maneuver, descent, landing, and required reserve are modeled as constant numbers. An air refueling track is represented along with the effects of a constant wind. The necessity of a refueling alternate is also included. The method of Sequential Quadratic Programming was used to obtain numerical results. Comparison of model results with a computer-generated flight plan shows that substantial fuel savings and a large increase of allowable cabin load are possible through the selection of an optimal rendezvous point and initial fuel combination. However, minimization of the mission fuel cost is only appropriate in situations where the cargo weight is a fixed quantity less than the maximum capacity of the aircraft. Otherwise, the cargo load should be maximized as this reduces the total number of missions required, and thus provides the greatest savings.

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